Cross-Border Mergers and Greenfield Foreign Direct Investment*

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Abstract

I present a model of international trade and foreign direct investment (FDI), where FDI is comprised of greenfield FDI and mergers and acquisitions (M&A). Working in a monopolistically competitive environment, merging firms do not reduce competition. Mergers are motivated by efficiency gains and transfer of technology and expertise. Following empirical evidence, I model greenfield investors as the more productive group relative to M&A firms. The model has two symmetric countries and generates two-way flows of both M&A and greenfield FDI. Greater proximity to a market makes more firms choose greenfield FDI over M&A when investing there. Empirical evidence supports this result.

KEYWORDS: Foreign direct investment, mergers, acquisitions, greenfield, firm heterogeneity.
JEL: F12; F23; O41.

1 Introduction

Most of the horizontal foreign direct investment (FDI) literature describes FDI as the building of a production facility abroad (greenfield FDI). It explores the trade-off between the benefit of economies of scale of producing at home versus the benefit of producing abroad and foregoing the payment of the variable costs of trade (e.g. transportation and tariffs). The bulk of FDI actually belongs to M&A activity, over eighty percent in 1999 according

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to UNCTAD (2000), or according to Head and Ries (2008) for the years between 1987 and 2001, two thirds of total FDI. According to Gugler et. al. (2003) for the period 1981-1998, cross-border mergers as a share of all mergers were 10.6% in the US, 29.9% in the UK, 33.5% in continental Europe, 52.6% in Japan, 30.0% in Australia, New Zealand and Canada and 28.5% in the rest of the world.

In order to study the effect of policy on FDI, it is important to properly model its composition and firms' incentives to chose a particular mode of entry into foreign markets. The purpose of this paper is to model FDI not only as greenfield investments but also M&A. In the literature on FDI composition and trade, mergers are modeled in an oligopolistic setting as in Neary (2009), where the incentive to merge is based on strategic motives (merging firms reduce competition), exploiting complementarities among merging parties (firm headquarters with a specific entrepreneurial ability and a production facility with a separate productivity) in a monopolistically competitive market as in Nocke and Yeaple (2007) and (2008) or in an oligopolistic market as in Norbäck and Persson (2007) and (2008). The current model suggests a different incentive for firms to merge: transfer of technology and managerial expertise from the more productive firm to the less productive one. There are three empirical regularities related to FDI that the model fits: first, greenfield investors are more productive than M&A firms. Second, the model generates two-way flows of both M&A and greenfield FDI. Third, the closer are the two countries, the more greenfield FDI is chosen over M&A as a mode of entry.

I build an endogenous growth model with an expanding variety of products and firms with heterogeneous productivities. There are two symmetric economies Home and Foreign. When a firm is "born" it draws a marginal cost from an exogenous distribution. Depending on how productive it turns out to be, it has several options to choose from: (i) to not enter any market, (ii) to enter only its local market, (iii) to enter its local market and to export to Foreign, (iv) to enter its local market and to merge with (take over) a firm abroad, or (v) to enter its local market and to invest in a new plant in Foreign that will allow it to produce its product abroad. Each of those choices are optimal depending on where on the productivity distribution a firm is. I solve the model for an equilibrium where the least productive firms choose (i), the more productive choose (ii), ... and the most productive choose option (v).

This ordering is certainly not arbitrary. Empirical evidence shows that exporters are more productive than non-exporters (see Bernard and Jensen (1999), Aw, Chung and Roberts (2000) and Clerides, Lach and Tybout (1998)), firms engaging in FDI are in turn more productive than exporting firms (see Girma, Kneller and Pisu (2005), Helpman (2006)) and within the group of firms choosing FDI as an option for entering the foreign market, the more productive ones are involved in greenfield FDI (see Nocke and Yeaple (2008)). The ordering is also supported by Raff et. al. (2011) who look at Japanese firm-level data.

In line with the theoretical literature on trade and firms with heterogenous productivities (in particular Melitz (2003) and Helpman, Melitz and Yeaple (2004)), I connect the choice to enter a market (both local and foreign) with a one-time payment of a fixed cost[1]. The

[1]See also Gustafsson and Segerstrom (2010) and Baldwin and Robert-Nicoud (2008) for treatments of the subject in endogenous growth settings.

magnitude of the fixed costs determines the productivity necessary to enter or not and if yes how (choices (i) through (v)). Helpman, Melitz and Yeaple (2004) have in addition to the usual fixed costs (for entering the local market and for exporting) a third one. This is the fixed cost for building a plant abroad. The innovative aspect of this paper is to introduce one more fixed cost: for merging with a foreign firm. Once the merger is completed, the home investor can use the production facilities of the foreign firm. A home firm can therefore enter the foreign market in one of three ways, by exporting, by merging with (acquiring) a foreign firm or by building a plant there.

The second innovative aspect of this model is how a merger is described. A home firm that chooses to acquire a foreign firm will be able to use that foreign firm's production facilities. The key here is that the Home firm's higher productivity will partially "transfer" to the foreign firm. I write partially because the foreign plant's productivity will be between the productivities of the two firms participating in the merger. When Renault took a third ownership in Nissan in 1999, it installed one of its top managers, Carlos Ghosn, as Nissan's CEO. He restructured Nissan and brought it back to profitability. It is this transfer of expertise and technology that I model. According to Bloom and Van Reenen (2010) management practices are an important source of firm productivity, where firms with better management are more productive and larger. In addition, in a study of M&A activity in Canada and the US around the time of the Canada-United States Free Trade Agreement from 1989, Breinlich (2008) finds that acquiring firms are bigger, more productive and more profitable in comparison to target firms.

Firms are born and draw a marginal cost before they choose to enter any market. Home firms acquire some of the failing foreign firms that would otherwise not start production due to too high marginal costs. After the foreign failing firm is acquired, it obtains a new productivity which depends on its productivity and that of the acquiring firm. Therefore a firm looking for a takeover target would prefer to merge with the most productive Foreign failing firm. I assume that the productivity of each firm is not known. What is known is only on which markets it is present or whether it is about to exit. A Home firm looking for a takeover target will pay the fixed cost for the merger and will be randomly assigned to one within the group of exiting Foreign firms. The benefit from the merger is split between the acquiring and the target firm.

Given this setup, I solve for a symmetric steady state equilibrium. I find in line with the existing literature that lower variable costs to trade allow for more firms to become exporters. In my model there are two more productivity thresholds, one that separates exporters from firms involved in M&A and the other the threshold dividing firms involved in M&A and firms that open their own factory abroad (greenfield FDI). Lower variable costs to trade make the productivity threshold for both greenfield FDI and M&A more stringent (less firms choose FDI as an option) and increase the share of greenfield investment in total FDI.

The next section lays out the model. Section three gives a solution and section four discusses the results. There is also an appendix where the more involving calculations are spelled out.

2 The Model

In the model there are two symmetric economies (countries) Home and Foreign, a single consumption good sector with Dixit-Stiglitz monopolistic competition and a single innovation sector where firms invest in R&D to create knowledge. Labor is the single factor of production and R&D. In each of the two economies, there is a fixed measure of households that provide labor services in exchange for wage payments. Each individual member of a household lives forever and is endowed with one unit of labor, which is inelastically supplied. Total population and labor supply in a country at time t is $L_t = L_0 e^{nt}$, where L_0 is the initial population and n the population growth rate. There is no unemployment in the economy. Firms invest in R&D to discover new varieties of products. This investment in R&D represents a one-time product development fixed cost. After the firm incurs that cost and discovers a product, it draws its marginal cost from a given distribution.

To enter the Home and Foreign markets, there are certain exogenously given fixed costs to be paid, all of them paid in terms of R&D labor. Every choice of entry is associated with the payment of a fixed cost while exporters in addition face iceberg trade costs when shipping goods to the foreign market. Instead of exporting, a firm can choose to take over a foreign firm and use its production facilities. The benefit for the acquiring firm is not only to be able to gain a foothold for its product on the other market, but it also transfers part of its productivity to the less productive foreign firm. Lastly, some firms choose instead of exporting or taking over a foreign firm to establish their own plant abroad.

2.1 Consumers

Households are infinitely lived and share identical preferences. Each household is modelled as a dynastic family that maximizes discounted lifetime utility

$$U \equiv \int_0^\infty e^{-(\rho-n)t} \ln(u_t) dt,$$

where $\rho > n$ is the subjective discount rate and u_t is the instantaneous utility of an individual at time t. The representative consumer has a CES utility function given by

$$u_t \equiv \left(\int_0^{m_t^c} x(\omega)^\alpha d\omega \right)^{\frac{1}{\alpha}},$$

where m_t^c is the measure of varieties available in the Home market, $x_t(\omega)$ is the amount an individual consumes of a particular variety ω at time t and the degree of differentiation between products is determined by $\alpha \in (0, 1)$. Products are gross substitutes with an elasticity of substitution $\sigma \equiv 1/(1-\alpha) > 1$.

Solving the static optimization problem gives the following demand function:

$$x(\omega) = \frac{p_t(\omega)^{-\sigma}}{P_t^{1-\sigma}} c_t, \tag{1}$$

where $P_t \equiv \left(\int_0^{m_t^c} p_t(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}$ is an aggregate price index, c_t is individual expenditure and $p_t(\omega)$ is the price of product ω at t. Taking prices and expenditure as given the solution to the intertemporal problem yields the familiar Euler equation $\dot{c}_t/c_t = r_t - \rho$, where r_t is the market interest rate. I solve for a steady state equilibrium with a constant consumer expenditure path, which is optimal only for $r_t = r = \rho$ for all t.

2.2 Innovation

Firms create knowledge by doing R&D. A unit of knowledge requires b_t units of labor for its production. Firms treat b_t as a parameter, but it changes with time due to knowledge spillovers. Following Jones (1995b), I assume $b_t = 1/(m_{Lt} + \lambda m_{Ft})^{\phi}$. The parameter $\phi < 1$ measures the strength of intertemporal knowledge spillovers. The parameter $\lambda \in [0, 1]$ measures the strength of international knowledge spillovers, where $\lambda = 0$ corresponds to no international spillovers and $\lambda = 1$ corresponds to perfect international spillovers. The number of varieties developed and produced in Home (not including products resulting from mergers or greenfield FDI) is m_{Lt} and m_{Ft} is the number developed and produced in Foreign. Given symmetry between the countries, I can write $m_{Lt} = m_{Ft} \equiv m_t$, and hence:

$$b_t = \frac{1}{(1+\lambda)^{\phi} m_t^{\phi}}. \tag{2}$$

For $\phi > 0$, researchers become more productive with time. For $\phi < 0$, R&D becomes more difficult. I follow Jones (1995b) in choosing $\phi < 1$ to rule out explosive growth. Taking logs and differentiating with respect to time yields $\dot{b}_t/b_t = -\phi \dot{m}_t/m_t$[2].

To create a new product variety, a firm needs to create F_I units of knowledge which means that it needs to invest $b_t F_I$ units of labor at time t. After the invention of a new variety, the firm draws a marginal cost parameter which indicates how many labor units it takes to produce a unit of the good. This marginal cost parameter does not change over time and is drawn from a Pareto distribution which has a probability density function $g(a)$ with support $[0, \bar{a}]$ and a cumulative density function $G(a) = \int_0^a g(a)da = (a/\bar{a})^k$. Melitz (2003) works with a general distribution. Helpman, Melitz and Yeaple (2004) show that the model becomes much more tractable if one chooses a Pareto distribution. The empirical literature on the size distribution of firms suggests that this is a reasonable choice (see Del Gatto, Giordano and Ottaviano (2006)). The model will generate several types of marginal cost thresholds which determine whether a firm enters a market or not and if it does, how, by exporting, acquiring a firm abroad or by building a new plant. I use at some places throughout the text productivity instead of marginal cost, keeping in mind that low marginal cost corresponds to high productivity.

[2]The choice of R&D function yields an economic growth rate dependent on the population growth parameter n, the elasticity of substitution σ and the R&D parameter ϕ, thus making the model one of semi-endogenous growth. Looking at US manufacturing industry data, Venturini (2010a,b) find that semi-endogenous growth models have better empirical support than fully-endogenous growth models.

2.3 Producers

Given a particular marginal cost draw $a(\omega)$ for producing the new variety ω, a firm makes the following profits selling in its local market:

$$\pi_{Lt} = \max_{p_L}(p_L - a(\omega))x_{Lt}(\omega),$$

where p_L is the price a firm holding the patent for product variety ω sets on its local market and $x_{Lt}(\omega)$ is demand for that locally manufactured product. Using (1) and $C_t \equiv c_t L_t$ as aggregate expenditure at t, I obtain that the optimal price is $p_L(\omega) = \frac{\sigma}{\sigma-1}a(\omega)$ and local profits are

$$\pi_{Lt} = \delta \left(\frac{a(\omega)}{P_t}\right)^{1-\sigma} C_t, \tag{3}$$

where $\delta \equiv (\sigma-1)^{\sigma-1}\sigma^{-\sigma}$. A firm makes the following profits selling in its export market:

$$\pi_{Et} = \max_{p_E}(p_E - \tau a(\omega))x_{Et}(\omega),$$

where $\tau > 1$ is an iceberg variable cost to trade and $x_{Et}(\omega)$ is demand for an exported product ω. Optimization yields $p_E(\omega) = \frac{\sigma}{\sigma-1}\tau a(\omega)$ and exporting profits are

$$\pi_{Et} = \delta \left(\frac{\tau a(\omega)}{P_t}\right)^{1-\sigma} C_t.$$

I can express the relation between profits from selling on the local market and from exporting as $\pi_{Et} = \theta\pi_{Lt}$, where $\theta \equiv \tau^{1-\sigma}$. The case of autarky corresponds to $\theta = 0$ and free trade to $\theta = 1$.

2.4 Value Equations and Marginal Cost Cutoffs

There are four types of firms. This first type is those that sell only at home, their value will be denoted by $v_L(a)$. The second type is those that sell at home and export, with value $v_L(a) + v_E(a)$, where $v_E(a)$ is the value of the exporting section of a firm's operation. For brevity I will suppress the time subscript in the value functions and profits. There also are firms that sell at home and have merged with a foreign firm. They have value $v_L(a)+v_L(a')$, where a' is the productivity of the foreign plant and is a function of the productivities of the two merging firms (a' will be formally defined shortly). The fourth type is those firms that sell at Home and have a subsidiary abroad. They have value $2v_L(a)$, since they sell one product on two markets without paying any variable costs to trade.

Looking at $v_L(a)$ first, I must have that the return on an equity claim in a firm (profits plus the change in its value for a short period dt) be equal to the riskless rate of return in the economy r. There is no risk from investing in a firm whose productivity is already known, hence $rv_L(a)dt = \pi_L(a)dt + \dot{v}_L dt$. Solving yields:

$$v_L(a) = \frac{\pi_L(a)}{r-g}, \tag{4}$$

where $g \equiv \dot{v}_L/v_L$. The value equation for the exporting section of a firm must satisfy $rv_E(a)dt = \pi_E(a)dt + \dot{v}_E dt$. The intuition is identical to the one for a firm's local operation. Solving, I obtain

$$v_E(a) = \frac{\pi_E(a)}{r-g}. \tag{5}$$

From (4), (5) and $\pi_{Et} = \theta\pi_{Lt}$, one can see that $v_E(a) = \theta v_L(a)$ and $g \equiv \dot{v}_L/v_L = \dot{v}_E/v_E$.

Let the value function from selling in the local market net of the fixed cost of entering the local market be $f_L(a) \equiv v_L(a) - b_t F_L$. Let the marginal cost below which firms find it optimal to enter their local market be a_L. Firms with marginal cost draws of $a \in (a_L, \bar{a})$ will not be able to cover the fixed cost for entering the local market $b_t F_L$ and will therefore not enter. The value of the firm with the threshold marginal cost net of the fixed cost to entering the local market must equal the value of a failing firm for now written $v_F(a_L)$:

$$f_L(a_L) = v_F(a_L).$$

2.4.1 Mergers and Acquisitions

An innovative aspect of my model is the M&A process and its benefits. A more detailed support for the M&A assumptions that I make is therefore necessary. The industrial organization literature has emphasized two main motives for a merger: efficiency gains and strategic motives. By strategic motives one has in mind reducing competition in a market where firms are not atomistic and affect the behavior of others. In my model with monopolistic competition, each firm is infinitely small and its merger with another firm does not affect the behavior of other firms. Without dismissing the importance of strategic interactions between firms in oligopolistic markets, I focus my attention on efficiency gains through transfer of knowledge and study this as one of the possible channels through which variable costs to trade can affect the composition of FDI.

Here are the most important assumptions I make regarding the M&A process:

Assumption 1: The acquiring firm pays a fixed cost to initiate a merger.

The fixed cost can be seen as a fee for a consultant to evaluate and facilitate the merger, the cost of restructuring the foreign enterprise and facilitating its entry in the foreign market.

Assumption 2: The acquiring firm can merge with a failing foreign firm (the failing firms are those with $a \in (a_L, \bar{a})$)

A great number of firm mergers in Eastern Europe in the 1990s for instance were negotiated to save failing state enterprises. As part of a privatization process, the governments were looking for foreign investors, which had the capability to increase those failing firms' productivities and to save them from bankruptcy.[3]

Assumption 3: There are gains from a merger.

Jensen (1988) cites empirical evidence from the M&A literature, that takeovers "generate substantial gains: historically, eight percent of the total value of both companies."

[3] The failing firms can also be described as plants with low productivity that belong to larger firms consisting of several plants, each with its own unique productivity. In this case mergers could be seen as a part of the process of firms buying and selling corporate assets.

Assumption 4: The gain from a merger comes from two sources i) the acquired firm obtains a lower marginal cost of production (efficiency gains, can be seen as technology or knowledge transfer between firms) ii) the acquiring firm gains a foothold in the foreign market for its product and uses the acquired firm's production plant abroad. The foreign product for which the foreign firm has a patent is not produced.

According to Jensen (1988), empirical studies show that the gains from the merger represent "gains to economic efficiency, not redistribution between various parties". Mandelker (1974) finds evidence that mergers can be seen as a mechanism through which the market replaces incompetent management, thus increasing efficiency. Conyon et. al. (2002) find that firms that are acquired by foreign companies show an increase in labor productivity of 13%.

Turning to the theoretical literature, some papers focus on modeling efficiency gains as reductions in marginal cost for the post-merger firm (see Werden (1996)). Roller et. al. (2006) provide a useful discussion and summary of the theoretical literature on the role of efficiencies in M&A and how they are modeled. They talk about five different sources of efficiency gains and say that *all five* can be modeled either as a reduction to variable or to fixed costs. The first source of gains is cost savings from reallocating production. There is reallocation of production in my model in the sense that a home firm gains a foothold in the foreign market using the other firm's production facilities. Another source is economies of scale. Roller et. al. (2006) further mention technological progress, modeled either as cost reduction or product quality improvement. They specifically mention diffusion of know-how as a way to model technological progress. My approach fits this description best, since I have the less productive firm 'learn' from the more productive one. The last two channels of efficiency gains described in the theoretical literature are savings in factor prices such as intermediate goods or the cost of capital (not present in my model) and reduction of slack (managerial and X-efficiency). My approach can fit the last one since the transfer of productivity from the more efficient to the less efficient firm can be seen as replacement of incompetent management.

Assumption 5: The gain from the merger is split between the acquiring and the target firm as a result of bargaining.

If the target firm leaves the negotiation, it does not have the option to wait for another match and exits immediately. If the acquiring firm leaves the negotiation, it will not be matched to another failing firm. The outside options of both firms are therefore zero. The Nash solution to the bargaining problem assigns a share $0 < \psi < 1$ of the gains from the merger to the acquiring firm and a share $1 - \psi$ to the target firm, where the parameter ψ represents the bargaining strength of the acquiring firm.

When a firm invests to merge with another firm abroad, it will pay the fixed cost $b_t F_M$, where F_M is the fixed cost for initiating a merger in terms of R&D units, and will be randomly assigned to a firm within that group. The precise productivity of the firm with which the acquiring firm is matched is not known when the investment $b_t F_M$ is made. What is known is whether the firm is failing or not, and if not, on which market it sells. After the fixed cost for initiating the merger has been paid and firms have been matched, productivities are revealed and the two firms enter negotiations on how to split the proceeds from the merger.

Let a_h be the marginal cost of a home firm and $a_f \in (a_L, \bar{a})$ that of the foreign failing firm. In the equilibrium for which I solve and for which conditions are provided below, the failing firms are the ones with highest marginal costs, therefore $a_h < a_f$. The restructured foreign enterprise will have a marginal cost

$$a' \equiv (a_f/\mu)a_h,$$

where μ is the mean of the Pareto distribution.

This restructuring technology has the desirable property that a' is an increasing function of both a_f and a_h. The higher is the marginal cost of the foreign failing firm or of the home acquiring enterprise, the higher is the marginal cost of the post-merger foreign plant. In addition, the restructured plant will have a marginal cost in between the marginal costs of the acquiring and acquired firms, that is $a_h < a' < a_f$. To guarantee that this inequality holds, I solve the model for a steady state equilibrium satisfying

$$a_M < \mu < a_L. \tag{6}$$

This implies that $\mu < a_f$ for all a_f, also $a_h < \mu$ for all a_h, from which follows that $a_h < a' < a_f$.

Next, I define the net benefit of entering the foreign market in the three possible ways: through exporting, acquiring a foreign firm or building a plant abroad. The function $f_E(a) \equiv \theta v_L(a) - b_t F_E$ represents the value of exporting net of the fixed cost of entering the export market. The function $f_G(a) \equiv v_L(a) - b_t F_G$ represents the benefit of greenfield FDI net of the fixed cost of building a plant abroad. The function $f_M(a) \equiv \psi E_f\left[v_L(a')\right] - b_t F_M$ represents the expected benefit of a merger net of the fixed cost for initiating the merger. In the $f_M(a)$ function, the expectation takes into consideration that a_f, from the perspective of an acquiring firm, is a random draw from the truncated Pareto distribution with a probability density function $g(a)/(1 - G(a_L))$. Remember that $G(a_L)$ is the probability that $a \leq a_L$, so $1 - G(a_L)$ is the probability that $a > a_L$ and it is always the case that $a_f > a_L$ since only failing firms are acquired. Since a' depends on both a_f and a_h, I write the expectations operator E_f in order to signify that expectations are taken with regard to a_f. Later in the text when the expectation is taken with regard to a_h, I will write E_h.

Equations (3) and (4) imply that:

$$\begin{aligned} v_L(a') &= \frac{\pi_L(a')}{r-g} \\ &= \frac{\delta P_t^{\sigma-1} C_t}{r-g} (a')^{1-\sigma} \\ &= \frac{\delta P_t^{\sigma-1} C_t}{r-g} \left((a_f/\mu)a_h\right)^{1-\sigma} \\ &= (a_f/\mu)^{1-\sigma} v_L(a_h). \end{aligned}$$

Therefore

$$\begin{aligned} E_f\left[v_L(a')\right] &= E_f\left[(a_f/\mu)^{1-\sigma} v_L(a_h)\right] \\ &= v_L(a_h) E_f\left[(a_f/\mu)^{1-\sigma}\right]. \end{aligned}$$

In the appendix, I solve for the expected value $\xi \equiv E_f\left[(a_f/\mu)^{1-\sigma}\right]$ and show that ξ is a function of a_L and exogenous variables. I assume that $k > \sigma - 1$ to make sure that ξ is positive. I write $E_f\left[v_L(a')\right] = \xi v_L(a_h)$ and $f_M(a) \equiv \psi\xi v_L(a_h) - b_t F_M$. I can also write

$$\begin{aligned} E_h\left[v_L(a')\right] &= E_h\left[(a_h/\mu)^{1-\sigma} v_L(a_f)\right] \\ &= v_L(a_f) E_h\left[(a_h/\mu)^{1-\sigma}\right]. \end{aligned}$$

I define $\kappa \equiv E_h\left[(a_h/\mu)^{1-\sigma}\right]$. The assumption $k > \sigma - 1$ guarantees a positive κ. I can therefore write $E_h\left[v_L(a')\right] = \kappa v_L(a_f)$.

All firms that hold a patent before they draw a marginal cost at t have mass $\dot{m}_t/G(a_L)$. Of those only $1 - G(a_L)$ fail to enter their local market. Therefore the mass of all failing firms at t that have marginal cost $a \in \left(a_L, \bar{a}\right)$ (takeover targets) is $\dot{m}_t\left(1 - G(a_L)\right)/G(a_L)$. The mass of all firms discovering a product at t that have marginal cost within the range $a \in (a_G, a_M)$ (looking for a takeover target) is $\dot{m}_t\left(G(a_M) - G(a_G)\right)/G(a_L)$. Therefore the probability of being taken over is $\epsilon \equiv \frac{G(a_M) - G(a_G)}{1 - G(a_L)}$.

The value of a failing firm equals the likelihood with which that firm will become a takeover target times the share from the gains from a merger, times the expected gain:

$$\begin{aligned} v_F(a_f) &= (1-\psi)\epsilon E_h\left[v_L(a')\right] \\ &= (1-\psi)\epsilon\kappa v_L(a_f). \end{aligned}$$

In the appendix, I show that $\epsilon\kappa$ is a function of a_L and exogenous variables.

In order to find a_L I go back to $f_L(a_L) = v_L(a_L) - b_t F_L = v_F(a_L)$. As is shown in the appendix, substituting for the value function from (4) and then for profits from (3) yields

$$a_L^{1-\sigma} = \frac{F_L}{1 - (1-\psi)\epsilon\kappa} \frac{b_t(r-g)}{\delta P_t^{\sigma-1} C_t}. \tag{7}$$

Note that v_L is proportional to $a^{1-\sigma}$. The functions f_E, f_M and f_G are all defined as functions of a but when graphing these functions, it is convenient to think of them as functions of $a^{1-\sigma}$, since they are all upward-sloping and linear in $a^{1-\sigma}$. In all three functions, marginal cost a enters only through the term $a^{1-\sigma}$, which can be seen as a measure of productivity (marginal cost raised to a negative power).

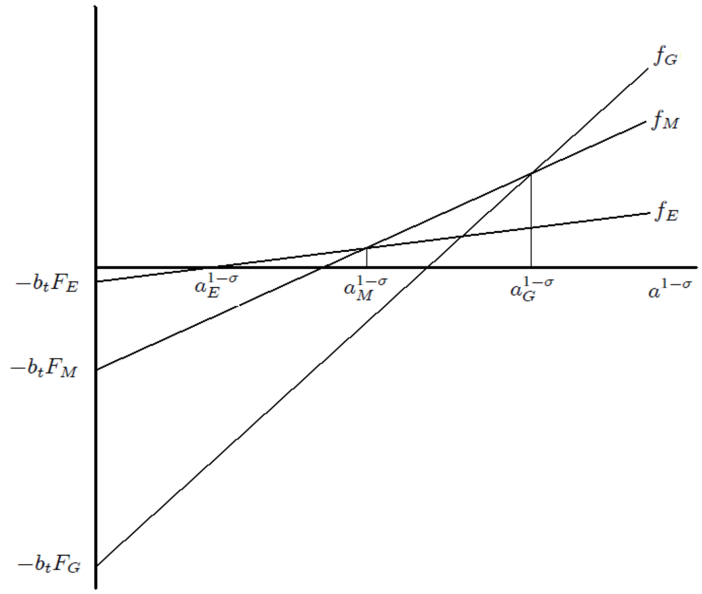

Figure 1.

The functions f_E, f_M and f_G are illustrated in Figure 1 and are drawn so that the firms with lowest marginal cost choose greenfield FDI ($a \in (0, a_G)$ or $a_G^{1-\sigma} < a^{1-\sigma}$), firms with slightly higher marginal costs would rather acquire a foreign failing firm ($a \in (a_G, a_M)$ or $a_M^{1-\sigma} < a^{1-\sigma} < a_G^{1-\sigma}$) and the least productive of firms entering the foreign market choose to export ($a \in (a_M, a_E)$ or $a_E^{1-\sigma} < a^{1-\sigma} < a_M^{1-\sigma}$). This is not the only possible equilibrium for which one can solve, but it is the one I am interested in, in order to fit the empirical evidence on firm productivity and preferable mode of entry into foreign markets cited in the introduction. For the ordering of outcomes illustrated in Figure 1 to occur, I need to assume that

$$F_E < F_M < F_G. \tag{8}$$

Also the slope of f_G must be steeper than that of f_M, which in turn must be steeper than that of f_E. For that to hold, I must have that

$$\theta < \psi\xi < 1. \tag{9}$$

The value of the foreign operation of an exporter with the cutoff marginal cost for entering the foreign market (denoted by a_E) must be equal to the fixed cost that it needs to pay to enter $f_E(a_E) = \theta v_L(a_E) - b_t F_E = 0$, as in Melitz (2003). Substituting for the value function from (4) and for profits from (3) yields

$$a_E^{1-\sigma} = \frac{F_E}{\theta} \frac{b_t(r-g)}{\delta P_t^{\sigma-1} C_t}. \tag{10}$$

As illustrated in Figure 1, the value from entering through a merger is lower than from entering as an exporter for less productive firms ($a^{1-\sigma} < a_M^{1-\sigma}$) but becomes preferable for more productive firms ($a^{1-\sigma} > a_M^{1-\sigma}$). I define a_M to be the marginal cost threshold where $f_E(a_M) = f_M(a_M)$. I substitute into this expression for the value functions and for profits from (3) and (4) to obtain

$$a_M^{1-\sigma} = \frac{F_M - F_E}{\psi\xi - \theta} \frac{b_t(r-g)}{\delta P_t^{\sigma-1} C_t}. \tag{11}$$

A similar argument goes for determining the threshold marginal cost separating the firms that choose to enter with a merger from those that built their own plant abroad. For higher marginal cost values firms would prefer to enter by means of a merger, but the firms with lowest marginal cost find it more profitable to enter by greenfield FDI. Let's call a_G the marginal cost cutoff where $f_G(a_G) = f_M(a_G)$. Substituting for the value functions and profits yields

$$a_G^{1-\sigma} = \frac{F_G - F_M}{1 - \psi\xi} \frac{b_t(r-g)}{\delta P_t^{\sigma-1} C_t}. \tag{12}$$

The formal derivation of all marginal cost cutoffs is provided in the appendix. I solve for an equilibrium where

$$0 < a_G < a_M < a_E < a_L < \bar{a} \tag{13}$$

holds. As I show in the appendix, in addition to (8) and (9), the following conditions must be satisfied for (13) to hold:

$$\frac{F_L}{1 - (1-\psi)\epsilon\kappa} < \frac{F_E}{\theta} \tag{14}$$

$$F_M > \psi\xi \frac{F_E}{\theta} \tag{15}$$

$$\frac{F_G - F_M}{F_M - F_E} > \frac{1 - \psi\xi}{\psi\xi - \theta}. \tag{16}$$

Condition (14) is similar to the one in Melitz (2003) ensuring that the more productive firms self-select into becoming exporters. At an intuitive level, it is reasonable to assume that a firm needs to pay a higher fixed cost for entering a foreign market than for entering its local market. Inequality (15) is more restrictive than $F_M > F_E$ in (8). F_M has to be sufficiently larger than F_E. This is a reasonable assumption, meaning that it is significantly more costly to negotiate a merger than to enter the foreign market as an exporter. Condition (16) says that F_G has to be sufficiently larger than F_M, or in other words, the cost to build a plant abroad must be sufficiently higher than the cost of negotiating a merger.

There is one more constraint that I impose on the exogenous parameters in the model. When I solve for the steady state equilibrium, I make sure that

$$0 < \epsilon < 1 \tag{17}$$

holds.

2.5 Innovation Incentives

To determine the incentive of firms to develop varieties, the benefit of innovating a product must be equal to the cost. The cost is F_I R&D units times the labor required to produce them b_t. The expected benefit is a bit more involving to describe.

First, upon drawing an unfavorable marginal cost $a > a_L$, the firm becomes a takeover target with a probability ϵ and gains a share $1-\psi$ from the proceeds of the merger. Second, given the firm draws a marginal cost low enough to enter its local market $a < a_L$, there is the expected benefit of selling there after paying the fixed cost to enter $b_t F_L$. Third, for a marginal cost within the range $a \in (a_M, a_E)$, in addition to selling in its local market, the firm pays a fixed cost $b_t F_E$ and starts exporting. Fourth, for a marginal cost within the range $a \in (a_G, a_M)$, the firm pays the fixed cost $b_t F_M$ and merges with a foreign failing firm, obtaining in the process a share ψ from the gains of the merger. Lastly, for a marginal cost $a \in (0, a_G)$, the firm pays a fixed cost $b_t F_G$ and builds a plant abroad. The benefits are summarized on the right-hand side of the equation below:

$$\begin{aligned} b_t F_I &= \int_{a_L}^{\bar{a}} \epsilon(1-\psi) E_h \left[v_L(a')\right] g(a_f) da_f \\ &+ \int_0^{a_L} \left(v_L(a) - b_t F_L\right) g(a) da + \int_{a_M}^{a_E} \left(\theta v_L(a) - b_t F_E\right) g(a) da \\ &+ \int_{a_G}^{a_M} \left(\psi E_f \left[v_L(a')\right] - b_t F_M\right) g(a_h) da_h + \int_0^{a_G} \left(v_L(a) - b_t F_G\right) g(a) da. \end{aligned}$$

The first integral represents the gain from a merger to a failing firm times the probability of such an event. The second integral shows the benefit from selling in the local market minus costs for entering it. The third, fourth and fifth integrals describe the benefits from entering the foreign market (net of fixed costs) depending on the firm's chosen mode of entry. I group the fixed costs on the left-hand side, divide both sides by $G(a_L)$, substitute for profits and use the definitions

$$F_x \equiv \frac{F_I}{G(a_L)} + F_L + F_E \frac{G(a_E) - G(a_M)}{G(a_L)} + F_M \frac{G(a_M) - G(a_G)}{G(a_L)} + F_G \frac{G(a_G)}{G(a_L)} \tag{18}$$

and

$$\begin{aligned} \Delta &\equiv \int_0^{a_L} a^{1-\sigma} \frac{g(a)}{G(a_L)} da + \theta \int_{a_M}^{a_E} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \\ &+\xi \int_{a_G}^{a_M} a_h^{1-\sigma} \frac{g(a_h)}{G(a_L)} da_h + \int_0^{a_G} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \end{aligned}$$

to obtain

$$b_t F_x = \frac{\delta P_t^{\sigma-1} C_t}{r-g} \Delta. \tag{19}$$

This is the innovation incentives condition. The left-hand side of (19) can be seen as the cost of developing a variety and the right-hand side as the benefit.

2.6 Solving for the Aggregate Price Index

I continue with finding an expression for the aggregate price index within a country P_t, which satisfies $P_t^{1-\sigma} = \int_0^{m_t^c} p_t(\omega)^{1-\sigma} d\omega$.

$$\begin{aligned} P_t^{1-\sigma} &= \int_0^{a_L} p_L(a)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da \\ &+ \int_{a_M}^{a_E} p_E(a)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da \\ &+ \int_{a_G}^{a_M} \left(\int_{a_L}^{\bar{a}} p_L(a')^{1-\sigma} \frac{g(a_f)}{1-G(a_L)} da_f \right) m_t \frac{g(a_h)}{G(a_L)} da_h \\ &+ \int_0^{a_G} p_L(a)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da, \end{aligned}$$

where $g(a)/G(a_L)$ is the steady state density function conditional on entry. To explain the price index I start with the first line. Those are the prices of all local originating firms with a productivity $a \in (0, a_L)$. Foreign originating firms with productivity $a \in (a_M, a_E)$ export to home and sell at $p_E(a)$. Their contribution to the price index is on line two. Line three describes the prices of all foreign firms that have merged with a home failing firm. Prices charged by those firms are based on a'. Line four describes the contribution to the price index of foreign firms that have a subsidiary at home. Substituting for prices, rearranging and then substituting for Δ yields:

$$P_t^{1-\sigma} = m_t \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \Delta. \tag{20}$$

2.7 Steady State Labor Market Clearing

Labor is inelastically supplied and mobile between sectors. All workers are employed either in the R&D sector, a total of L_{It}, or in the production sector, a total of L_{Pt}. Total labor supply can be expressed as $L_t = L_{Pt} + L_{It}$. Each worker is endowed with one unit of labor and receives a wage $w = 1$ per unit of labor supplied. Labor markets are perfectly competitive.

Total workforce in production is given by the sum of labor producing for the local market and labor producing for the foreign market. For brevity, I write $a(\omega)x_{Lt}(\omega)$ as ax_{Lt}. To produce a variety for the local market, a firm needs ax_{Lt} units of labor. To produce a variety for the export market, a firm needs τax_{Et} units of labor. Let $x_t'(\omega)$ be demand for a locally produced product with marginal cost $a'(\omega)$. I will for brevity write this demand as x_t'. Labor involved in production at t is

$$L_{Pt} = m_t \left(\begin{array}{c} \int_0^{a_L} ax_{Lt} \frac{g(a)}{G(a_L)} da + \int_{a_M}^{a_E} \tau ax_{Et} \frac{g(a)}{G(a_L)} da \\ + \int_{a_G}^{a_M} \left(\int_{a_L}^{\bar{a}} a'x_t' \frac{g(a_f)}{1-G(a_L)} da_f \right) \frac{g(a_h)}{G(a_L)} da_h + \int_0^{a_G} ax_{Lt} \frac{g(a)}{G(a_L)} da \end{array} \right).$$

The first integral expresses what is produced by all non-failing home originating firms for the local market. The second integral takes into consideration what is produced for exporting.

The third integral takes into consideration what is produced by the formerly failing local firms that were taken over by a foreign firm. The fourth integral takes into consideration the production of subsidiaries of foreign firms. I substitute for x_{Lt}, x_{Et}, x'_t and for Δ to obtain:

$$L_{Pt} = \frac{\sigma - 1}{\sigma} C_t.$$

The full employment condition $L_t = L_{Pt} + L_{It}$ implies that:

$$C_t = L_t + \frac{C_t}{\sigma} - L_{It}. \tag{21}$$

Aggregate income equals aggregate labor income L_t plus aggregate income from profits C_t/σ, minus wages paid in the innovation sector L_{It}.

I move on to labor dedicated to R&D activities. The labor dedicated to discovering a new product is $b_t F_I$. The mass of firms that discover a product at t is $\dot{m}_t/G(a_L)$. Total R&D labor cost for product innovation at t is therefore $\dot{m}_t b_t F_I/G(a_L)$. Of all firms that have discovered a product, of mass $\dot{m}_t/G(a_L)$, only a fraction $\int_0^{a_L} g(a)da = G(a_L)$ enter the local market and are productive enough to pay $b_t F_L$, hence the total R&D labor cost to the economy from entering the local market is $\dot{m}_t b_t F_L G(a_L)/G(a_L)$. Again, of all firms that have discovered a product only a fraction enter the foreign market and pay the $b_t F_E$ fixed cost. This fraction is $\int_{a_M}^{a_E} g(a)da = G(a_E) - G(a_M)$. Hence the cost paid by those firms is $\dot{m}_t b_t F_E \left(G(a_E) - G(a_M)\right)/G(a_L)$. A fraction $(G(a_M) - G(a_G))$ of all firms that have entered pay the fixed cost to take over a foreign firm $\dot{m}_t b_t F_M (G(a_M) - G(a_G))/G(a_L)$. A fraction $G(a_G)$ pay the fixed cost to invest in a foreign subsidiary $\dot{m}_t b_t F_G G(a_G)/G(a_L)$. The total amount of labor busy with R&D activities at time t is therefore:

$$\begin{aligned} L_{It} &= \dot{m}_t b_t \frac{F_I}{G(a_L)} + \dot{m}_t b_t F_L + \dot{m}_t b_t F_E \frac{G(a_E) - G(a_M)}{G(a_L)} + \\ &\quad + \dot{m}_t b_t F_M \frac{G(a_M) - G(a_G)}{G(a_L)} + \dot{m}_t b_t F_G \frac{G(a_G)}{G(a_L)}. \end{aligned}$$

From the definition of F_x it follows that

$$L_{It} = \dot{m}_t b_t F_x. \tag{22}$$

3 Solving the Model

I solve the model for a symmetric steady state equilibrium in which the endogenous variables have constant growth rates. To find the growth rate of varieties, I define $\gamma \equiv \dot{m}_t/m_t$. I substitute for $\dot{m}_t$ from (22) and then for b_t from (2) to obtain:

$$\gamma = \frac{L_{It}}{m_t^{1-\phi}} \frac{(1+\lambda)^{\phi}}{F_x}.$$

Since γ is constant in steady state, L_{It} must grow at the same rate as $m_t^{1-\phi}$, so $n = (1-\phi)\dot{m}_t/m_t$ and

$$\gamma = \frac{n}{1-\phi}.$$

To solve for a steady state equilibrium, I define the concept relative R&D difficulty $z_t \equiv \frac{m_t^{-\phi}}{L_t/m_t}$. From (2), $m_t^{-\phi}$ is a measure of absolute R&D difficulty. The greater $m_t^{-\phi}$ is, the more labor is needed to produce knowledge units. Combining profits from (3) with the price index from (20) yields the following expression for profits $\pi_{Lt} = \sigma^{-1}a^{1-\sigma}\Delta^{-1}c_t L_t/m_t$. It is clear that the relevant market size for variety ω is L_t/m_t. Firms are able to spread their R&D costs over a larger market for a higher L_t/m_t, therefore making R&D easier to finance. Relative R&D difficulty $z_t \equiv \frac{m_t^{-\phi}}{L_t/m_t}$ is R&D difficulty $m_t^{-\phi}$ relative to the size of the market L_t/m_t. Log-differentiating yields $\dot{z}_t/z_t = (1-\phi)\gamma - n = 0$. It follows that relative R&D difficulty is constant in steady state, that is, $z_t = z$ for all t.

I proceed with finding a_L. I substitute in (7) for the price index from (20), for b_t from (2), and for z to obtain:

$$\frac{\Delta}{a_L^{1-\sigma}} \frac{F_L}{1-(1-\psi)\epsilon\kappa} = \frac{(1+\lambda)^{\phi} C_t}{z(r-g)\sigma L_t}. \tag{23}$$

In the innovation incentives condition (19), I substitute for the the price index from (20), for b_t from (2), and for z to obtain

$$F_x = \frac{(1+\lambda)^{\phi} C_t}{z(r-g)\sigma L_t}. \tag{24}$$

Next, using the definition of Δ and solving for the integrals, I can write Δ as a function of a_L

$$\Delta \equiv a_L^{1-\sigma}\eta q_1(a_L), \tag{25}$$

where $q_1()$ is a function of a_L defined in the appendix. Combining (23) and (24) yields $F_x = \Delta \frac{F_L}{1-(1-\psi)\epsilon\kappa} a_L^{\sigma-1}$. I substitute for Δ from (25) to obtain $F_x = \frac{\eta q_1(a_L) F_L}{1-(1-\psi)\epsilon\kappa}$ which is an expression for F_x in exogenous parameters and a_L. Further, from (18), substituting for the cumulative distribution functions yields a second expression for $F_x = q_2(a_L)$, where $q_2()$ is a function of a_L defined in the appendix. Combining the two expressions for F_x gives an equation, which I solve numerically to obtain a solution for a_L:

$$q_2(a_L) = q_1(a_L)\frac{\eta F_L}{1-(1-\psi)\epsilon\kappa}. \tag{26}$$

I can treat a_L as known from now on. Given a_L, I know Δ from (25), a_E, a_G and a_M from (7), (10), (11) and (12).

To find C_t using (21), I need to first find L_{It} and C_t/σ. I substitute for $\dot{m}_t$ from (22) in $\gamma \equiv \dot{m}_t/m_t$, for b_t from (2) and for $z \equiv m_t^{1-\phi}/L_t$ to arrive at an equation for R&D labor:

$$L_{It} = \frac{F_x z L_t}{(1+\lambda)^{\phi}}\gamma. \tag{27}$$

In the innovation incentives condition (19), I substitute for the price index from (20), for b_t from (2) and rewrite $m_t^{1-\phi} = zL_t$ from the definition of z to arrive at an expression for C_t/σ:

$$\frac{C_t}{\sigma} = (r-g)\,\frac{F_x z L_t}{(1+\lambda)^{\phi}}. \tag{28}$$

See the appendix for a detailed derivation. I substitute for L_{It} from (27) and for C_t/σ from (28) into the equation for aggregate expenditures (21) to obtain:

$$C_t = L_t\left(1 + \frac{(r-g)\,F_x z}{(1+\lambda)^{\phi}} - \frac{F_x z}{(1+\lambda)^{\phi}}\gamma\right). \tag{29}$$

To find relative R&D difficulty z, I substitute for C_t from (29) into the innovation incentives condition (24):

$$z = \frac{(1+\lambda)^{\phi}}{F_x\left((\sigma-1)(r-g)+\gamma\right)}. \tag{30}$$

This completes the solution of the model.

4 Results

Empirical evidence (Nocke and Yeaple (2008)) suggests that greater geographical proximity increases the share of greenfield investment in total FDI. In this section I show that for plausible parameter values the model generates the same result. Geographical proximity can be interpreted as lower transportation costs (lower τ). I first find the share of greenfield FDI in total FDI. At every point in time firms are born and make a decision about which markets to enter and how. There is a constant flow of resources towards M&A and greenfield activity. The value of resources dedicated to M&A at t is $b_t F_M \dot{m}_t\left(G(a_M) - G(a_G)\right)/G(a_L)$, where $\left(G(a_M) - G(a_G)\right)/G(a_L)$ is the share of all local entrants $\dot{m}_t$ engaged in M&A and $b_t F_M$ is how much each firm that invests pays for a merger. Similarly, the total value of greenfield FDI is $b_t F_G \dot{m}_t G(a_G)/G(a_L)$, where $G(a_G)/G(a_L)$ is the share of all local entrants $\dot{m}_t$ that choose to build a plant abroad and $b_t F_G$ is the investment made by each of those firms. The share of greenfield investment, denoted by Ω, is measured by greenfield FDI value divided by the sum of greenfield FDI and M&A value:

$$\Omega \equiv \frac{F_G G(a_G)}{F_G G(a_G) + F_M\left(G(a_M) - G(a_G)\right)}.$$

It will be useful to see what happens with total FDI as well. To measure that I add the values of M&A and greenfield FDI. Since I am interested only in the direction of change of FDI as a result of lower variable costs to trade, I remove any multiplicative variables that do not depend on τ and evaluate

$$\mathcal{F} = z\frac{F_M G(a_M) + \left(F_G - F_M\right) G(a_G)}{G(a_L)}.$$

In addition, I look at the share of firms that export, which is $\chi \equiv (G(a_E) - G(a_M))/G(a_L)$.

In my computer simulation, I use the following parameter values: $\rho = 0.04$, $\alpha = 0.714$, $k = 3.735$, $n = 0.014$, $\phi = 0.7189$, $\lambda = 0.7$, $\bar{a} = 10$, $F_I = 3$, $F_L = 0.8$, $F_E = 1$, $F_M = 5$, $F_G = 60$ and $\psi = 0.9$. The subjective discount rate ρ is chosen so that the steady state interest rate would match the long-term average in the data. The real interest rate in the model is both the risk-free interest rate as well as a measure of the average real return on the stock market. I set $\rho = 0.04$ in accordance with McGrattan and Prescott (2005). The rate of substitution between products is set at $\alpha = 0.714$. This choice results in an elasticity of substitution of $\sigma = 3.49$, within the bounds of the estimates in Broda and Weinstein (2006) and a 40% markup, within range of the evidence presented in Basu (1996) and Norrbin (1993). Eaton, Kortum and Kramarz (2007) use data on exports and domestic sales by French firms and find that $k/(\sigma - 1) = 1.5$. To match this evidence, given my choice of σ, I set the parameter of the Pareto distribution at $k = 3.735$.

Dixit and Stiglitz (1977) have shown that each consumer's instantaneous utility coincides with their real consumption expenditure $u_t = C_t/P_tL_t$. To measure economic growth, I evaluate $\dot{u}_t/u_t$. In $u_t = C_t/P_tL_t$, I substitute for aggregate expenditure from (29), for the price index from (20) and log-differentiate to obtain

$$\frac{\dot{u}_t}{u_t} = \frac{n}{(1-\phi)(\sigma-1)}.$$

I set the population growth rate parameter $n = 0.014$ to match the annual rate of world population growth between 1991 and 2000 according to the World Development Indicators (World Bank, 2003). Given n, σ and the expression for $\dot{u}_t/u_t$, I choose $\phi = 0.7189$ to guarantee that the steady state rate of economic growth is 2%, consistent with the average GDP per capita growth rate in the US between 1950 and 1990 reported in Jones (2005). The rate of international knowledge spillovers is assumed to be smaller than one, $\lambda = 0.7$. The maximum marginal cost a firm can draw, $\bar{a}$, is a scale parameter and I set it equal to 10.[4] Particular fixed cost values are chosen according to (14), (15), (16), (17), also making sure that after solving for a_L, I am left with $a_L < \bar{a}$ and that (6) holds[5]. The fixed cost for entering the local market F_L is smaller than the one for entering the foreign market as an exporter F_E, which in turn is smaller than the fixed cost for initiating a merger F_M. The most expensive mode of entry abroad is by building a plant F_G. I evaluate the model for a change in τ from 1.9 to 1.5 (or θ changing from 0.20 to 0.36). At low values of τ there would be no FDI and all firms would prefer to export. To comply with $\theta < \psi\xi < 1$ (inequality (9)), I can not choose very low values for ψ (the share of the M&A gains that go to the acquiring

[4]The specific choice of $\bar{a}$ is not important for the results of the model. To be precise, $\bar{a}^k$ is the scale parameter. An increase in $\bar{a}^k$, accompanied by a proportionate increase in fixed costs would result in higher marginal cost cutoff values, but would not change variables like χ, Ω etc.

[5]Condition (17) is $0 < \epsilon < 1$. From $\epsilon = \frac{(a_M/a_L)^k - (a_G/a_L)^k}{(\bar{a}/a_L)^k - 1}$, $k > 1$ and (13) follows that $0 < \epsilon$. What remains to be checked when choosing fixed cost values is only $\epsilon < 1$.

firm). As long as (9) holds, the choice of ψ is not critical for the results. I set it at $\psi = 0.9$.

τ	a_L	a_E	a_M	a_G	z	F_x	Δ	χ	ξ	$\epsilon\kappa$	Ω	$\digamma$
1.9	9.71	5.44	3.73	1.5532	1.38	4.43	0.0130	0.08	0.573	3.16	0.32	0.27
1.7	9.75	6.05	3.37	1.5496	1.39	4.39	0.0130	0.14	0.570	3.01	0.40	0.21
1.5	9.83	6.79	2.69	1.5400	1.41	4.33	0.0132	0.24	0.564	2.68	0.62	0.13

Table 1. The Effects of Lower Variable Costs to Trade ($\tau \downarrow$)

The results from solving the model numerically are shown in Table 1. By looking at the column for χ (percentage of firms that export), one can see that the majority of firms are non-exporters in equilibrium. This is what Bernard et. al. (2003) find in their study of $200,000$ US manufacturing plants, where only 21% report exporting. The share of greenfield investment in total FDI increases from 32% at $\tau = 1.9$ to 62% at $\tau = 1.5$. This share of greenfield FDI is consistent with data reported in Head and Ries (2008) (one third of total FDI). The share of workers involved in all types of R&D activities, L_{It}/L_t, can be obtained directly from the equation for R&D labor (27) and substituting for z from (30):

$$\frac{L_{It}}{L_t} = \frac{\gamma}{(\sigma - 1)(r - g) + \gamma}.$$

The ratio does not depend of variable costs to trade and given the choice of parameters, it is $L_{It}/L_t = 0.2083$ or 20%.

Lower transportation costs (lower τ) lead to a greater share of greenfield FDI in total FDI (higher Ω). The intuition is not immediately obvious. Lower variable costs to trade make the exporting option more preferable. This is known in the literature as the proximity-concentration trade-off. The incentive to build or acquire a plant abroad decreases as it becomes cheaper to export. Total FDI decreases (last column of Table 1). This decrease happens at the expense of M&A activity. To better understand the forces at work, I spell out a_G and a_M. From (7), (11) and (12), I obtain:

$$a_G = a_L \left(\frac{1 - (1 - \psi)\epsilon\kappa}{1 - \psi\xi} \frac{F_G - F_M}{F_L} \right)^{\frac{1}{1-\sigma}}$$

$$a_M = a_L \left(\frac{1 - (1 - \psi)\epsilon\kappa}{\psi\xi - \theta} \frac{F_M - F_E}{F_L} \right)^{\frac{1}{1-\sigma}}.$$

Contrary to Melitz (2003) lower variable costs to trade increase a_L. The decision of a firm to enter or exit its local market is based not only on its ability to pay the entry fixed cost but also on its exit value (becoming a takeover target), which equals the probability of a merger times the expected gain. Lower τ leads to greater competitive pressures thus decreasing a_L. Lower τ leads to a lower $\epsilon\kappa$ which makes exit less attractive (more firms want to enter the local market), thus increasing a_L. For the chosen parameter values the second channel is stronger.

Both a_G and a_M increase through a_L, meaning that more firms want to enter by means of FDI. The second channel through which lower variable costs to trade affect the marginal cost thresholds is $\epsilon\kappa$, the probability of being taken over times a measure of the expected productivity of an acquiring firm. Lower τ leads to lower $\epsilon\kappa$, thus decreasing both a_M and a_G. The third channel through which geographical proximity affects the marginal cost thresholds is ξ, a measure of the productivity of failing firms. By making the most productive exiting firms enter (increasing a_L), lower variable costs to trade decreases the expected productivity of failing firms. Lower ξ decreases the incentive for firms to enter by means of M&A, thus lowering a_M and increasing a_G. The fourth channel through which lower transportation costs affect the marginal cost thresholds (in this case only a_M) is by directly appearing in a_M through the θ term. Here we have the the proximity-concentration trade-off at play. Exporting becomes more attractive[6].

Given the chosen parameter values the second, third and fourth channels are stronger than the first and lead to a lower a_M, while the second channel is stronger than the first and third and leads to a lower a_G. The decrease in a_M is greater than that in a_G thus leading to a higher share of greenfield FDI in total FDI, Ω.

5 Conclusion

I develop a model of international trade and foreign direct investment, where FDI consists of cross-border mergers and greenfield FDI. I abstract from any strategic motives for a merger, since I work with firms in a monopolistically competitive environment. The incentive for firms to merge comes from the transfer of technology and managerial know-how. Firms have heterogeneous productivities and the model has steady state endogenous growth. Exporters are more productive than non-exporters. Firms that engage in FDI are more productive than exporters and within the group of FDI firms, it is the most productive ones that become greenfield investors.

In addition to the unique approach to how M&A is modelled, the contribution of the current setup is twofold: first, both greenfield FDI and cross-border M&A exist simultaneously and go both ways from Home to Foreign and from Foreign to Home. This is not present in Nocke and Yeaple (2008), which is the model closest to mine when it comes to the results on FDI composition it generates. In their model of asymmetric countries, M&A flows both ways. Greenfield FDI however goes only from the richer to the poorer country, while as income differences between the two countries become smaller, greenfield FDI decreases. Given their setup, there would be no greenfield FDI between equally developed economies, which is clearly at odds with the evidence.

Second, greater proximity to the foreign market increases the share of greenfield FDI. In order to generate this result , Nocke and Yeaple (2008) assume that the fixed cost for opening up a factory abroad is lower, the smaller the distance to the foreign country. Given that my model has iceberg trade costs (not present in Nocke and Yeaple (2008)), I believe that

[6]Abstracting from changes in a_L, $\epsilon\kappa$ and ξ, $\tau\downarrow\Longrightarrow\theta\uparrow\Longrightarrow\psi\xi-\theta\downarrow\Longrightarrow\frac{F_M-F_E}{(\psi\xi-\theta)F_L}\uparrow\Longrightarrow a_M\downarrow$.

it generates that particular result in a more natural way, thinking of distance as affecting variable rather than fixed costs.

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Appendix

The Marginal Cost Cutoffs

In this section, I find the marginal cost cutoffs. To solve for a_L I use from $f_L(a_L) = v_L(a_L) - b_t F_L = v_F(a_L)$:

$$\begin{aligned} v_L(a_L) - b_t F_L &= (1-\psi)\epsilon\kappa v_L(a_L) \\ v_L(a_L)(1-(1-\psi)\epsilon\kappa) &= b_t F_L \\ v_L(a_L) &= \frac{b_t F_L}{1-(1-\psi)\epsilon\kappa} \end{aligned}$$

I substitute for $v_L(a_L)$ using (4):

$$\frac{\pi_L(a_L)}{r-g} = \frac{b_t F_L}{1-(1-\psi)\epsilon\kappa}$$

and for profits using (3):

$$\frac{\delta a_L^{1-\sigma} P_t^{\sigma-1} C_t}{r-g} = \frac{b_t F_L}{1-(1-\psi)\epsilon\kappa}.$$

Rearranging terms yields equation (7) in the main text:

$$a_L^{1-\sigma} = \frac{F_L}{1-(1-\psi)\epsilon\kappa} \frac{b_t(r-g)}{\delta P_t^{\sigma-1} C_t}.$$

To solve for a_E, I use $f_E(a_E) = \theta v_L(a_E) - b_t F_E = 0$ to obtain

$$\theta v_L(a_E) = b_t F_E.$$

I substitute for $v_L(a_E)$ from (4) to obtain

$$\frac{\pi_L(a_E)}{r-g} = b_t F_E/\theta,$$

and for profits using (3):

$$\frac{\delta a_E^{1-\sigma} P_t^{\sigma-1} C_t}{r-g} = b_t F_E/\theta.$$

Rearranging terms yields equation (10) in the main text:

$$a_E^{1-\sigma} = \frac{F_E}{\theta} \frac{b_t(r-g)}{\delta P_t^{\sigma-1} C_t}.$$

To solve for a_M, I use $f_M(a_M) = f_E(a_M)$:

$$\psi\xi v_L(a_M) - b_t F_M = \theta v_L(a_M) - b_t F_E$$

$$v_L(a_M) = \frac{b_t F_M - b_t F_E}{\psi\xi - \theta}.$$

I substitute for v_L using (4):

$$\frac{\pi_L(a_M)}{r-g} = \frac{b_t F_M - b_t F_E}{\psi\xi - \theta}$$

and for profits using (3):

$$\frac{\delta a_M^{1-\sigma} P_t^{\sigma-1} C_t}{r-g} = \frac{b_t F_M - b_t F_E}{\psi\xi - \theta}.$$

Rearranging terms yields equation (11) in the main text:

$$a_M^{1-\sigma} = \frac{F_M - F_E}{\psi\xi - \theta} \frac{b_t(r-g)}{\delta P_t^{\sigma-1} C_t}.$$

To solve for a_G I use $f_G(a_G) = f_M(a_G)$:

$$\psi\xi v_L(a_G) - b_t F_M = v_L(a_G) - b_t F_G$$

$$v_L(a_G) = b_t \frac{F_G - F_M}{1 - \psi\xi}.$$

I substitute for v_L using (4):

$$\frac{\pi_L(a_G)}{r-g} = b_t \frac{F_G - F_M}{1 - \psi\xi}$$

and for profits using (3):

$$\frac{\delta a_G^{1-\sigma} P_t^{\sigma-1} C_t}{r-g} = b_t \frac{F_G - F_M}{1 - \psi\xi}.$$

Rearranging terms yields equation (12) in the main text:

$$a_G^{1-\sigma} = \frac{F_G - F_M}{1 - \psi\xi} \frac{b_t(r-g)}{\delta P_t^{\sigma-1} C_t}.$$

For (13) to hold, the following three conditions must be satisfied: First, for $a_E < a_L$ or $a_L^{1-\sigma} < a_E^{1-\sigma}$, (7) and (10) imply that

$$\frac{F_L}{1 - (1-\psi)\epsilon\kappa} < \frac{F_E}{\theta}.$$

This is condition (14) in the main text. Second, for $a_M < a_E$ or $a_E^{1-\sigma} < a_M^{1-\sigma}$, (10) and (11) imply that

$$\frac{F_M - F_E}{\psi\xi - \theta} \frac{\theta}{F_E} > 1.$$

I multiply both sides by the denominator (from (9), it follows that the denominator is positive) to obtain:

$$\begin{aligned} (F_M - F_E)\,\theta &> (\psi\xi - \theta) F_E \\ \theta F_M - \theta F_E &> \psi\xi F_E - \theta F_E, \end{aligned}$$

and rearranging yields condition (15) in the main text:

$$F_M > \frac{\psi\xi}{\theta} F_E.$$

Third, for $a_G < a_M$ or $a_M^{1-\sigma} < a_G^{1-\sigma}$, (11) and (12) imply that

$$\frac{F_G - F_M}{1 - \psi\xi} \frac{\psi\xi - \theta}{F_M - F_E} > 1,$$

and rearranging yields condition (16) in the main text:

$$\frac{F_G - F_M}{F_M - F_E} > \frac{1 - \psi\xi}{\psi\xi - \theta}.$$

Finding ξ

$$\begin{aligned} \xi &\equiv E_f\left[(a_f/\mu)^{1-\sigma}\right] \\ &= \frac{1}{\mu^{1-\sigma}} \int_{a_L}^{\bar{a}} a_f^{1-\sigma} \frac{g(a_f)}{1 - G(a_L)} da_f. \end{aligned}$$

From $G(a) = (a/\bar{a})^k$ and $G'(a) = g(a)$, I obtain $g(a) = ka^{k-1}/\bar{a}^k$. I substitute for $G(a)$ and $g(a)$ to obtain:

$$\begin{aligned} \xi &= \frac{1}{\mu^{1-\sigma}} \int_{a_L}^{\bar{a}} a_f^{1-\sigma} \frac{ka_f^{k-1}/\bar{a}^k}{1 - (a_L/\bar{a})^k} da_f \\ &= \frac{1}{\mu^{1-\sigma}} \int_{a_L}^{\bar{a}} a_f^{k-\sigma} \frac{k}{\bar{a}^k - a_L^k} da_f \\ &= \frac{1}{\mu^{1-\sigma}} \left[a_f^{k-\sigma+1}\right]_{a_L}^{\bar{a}} \frac{1}{\bar{a}^k - a_L^k} \frac{k}{k - \sigma + 1} \\ &= \frac{\eta}{\mu^{1-\sigma}} \frac{\bar{a}^{k-\sigma+1} - a_L^{k-\sigma+1}}{\bar{a}^k - a_L^k}, \end{aligned}$$

where I assume that $k - \sigma + 1 > 0$. Note that ξ depends on a_L.

Finding $\epsilon\kappa$

From $\kappa \equiv E_h\left[(a_h/\mu)^{1-\sigma}\right]$ and $\epsilon \equiv \frac{G(a_M)-G(a_G)}{1-G(a_L)}$ follows:

$$\begin{aligned}
\epsilon\kappa &= \frac{G(a_M)-G(a_G)}{1-G(a_L)}\frac{1}{\mu^{1-\sigma}}\int_{a_G}^{a_M} a_h^{1-\sigma}\frac{g(a_h)}{G(a_M)-G(a_G)}da_h \\
&= \frac{(a_M/\bar{a})^k-(a_G/\bar{a})^k}{1-(a_L/\bar{a})^k}\frac{1}{\mu^{1-\sigma}}\int_{a_G}^{a_M} a_h^{1-\sigma}\frac{k a_h^{k-1}/\bar{a}^k}{(a_M/\bar{a})^k-(a_G/\bar{a})^k}da_h \\
&= \frac{1}{\mu^{1-\sigma}}\int_{a_G}^{a_M} a_h^{k-\sigma}\frac{k/\bar{a}^k}{1-(a_L/\bar{a})^k}da_h \\
&= \frac{1}{\mu^{1-\sigma}}\left[a_h^{k-\sigma+1}\right]_{a_G}^{a_M}\frac{1}{\bar{a}^k-a_L^k}\frac{k}{k-\sigma+1} \\
&= \frac{\eta}{\mu^{1-\sigma}}\frac{a_M^{k-\sigma+1}-a_G^{k-\sigma+1}}{\bar{a}^k-a_L^k}
\end{aligned}$$

where for brevity I write $\eta \equiv k/(k-\sigma+1)$ and assume that $k-\sigma+1>0$. Further

$$\begin{aligned}
\epsilon\kappa &= \frac{\eta}{\mu^{1-\sigma}}\frac{a_L^{k-\sigma+1}}{a_L^k}\frac{(a_M/a_L)^{k-\sigma+1}-(a_G/a_L)^{k-\sigma+1}}{(\bar{a}/a_L)^k-1} \\
&= \frac{\eta}{\mu^{1-\sigma}}a_L^{1-\sigma}\frac{(a_M/a_L)^{k-\sigma+1}-(a_G/a_L)^{k-\sigma+1}}{(\bar{a}/a_L)^k-1}
\end{aligned}$$

To find a_M/a_L, I use (7) and (11):

$$\begin{aligned}
\frac{a_M^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{\psi\xi-\theta}\frac{F_M-F_E}{F_L} \\
\frac{a_M}{a_L} &= \left(\frac{1-(1-\psi)\epsilon\kappa}{\psi\xi-\theta}\frac{F_M-F_E}{F_L}\right)^{\frac{1}{1-\sigma}}.
\end{aligned}$$

To find a_G/a_L, I use (7) and (12):

$$\begin{aligned}
\frac{a_G^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{1-\psi\xi}\frac{F_G-F_M}{F_L} \\
\frac{a_G}{a_L} &= \left(\frac{1-(1-\psi)\epsilon\kappa}{1-\psi\xi}\frac{F_G-F_M}{F_L}\right)^{\frac{1}{1-\sigma}}.
\end{aligned}$$

$$\epsilon\kappa = \frac{\eta}{\mu^{1-\sigma}}a_L^{1-\sigma}\frac{\left(\frac{1-(1-\psi)\epsilon\kappa}{\psi\xi-\theta}\frac{F_M-F_E}{F_L}\right)^{\frac{k-\sigma+1}{1-\sigma}}-\left(\frac{1-(1-\psi)\epsilon\kappa}{1-\psi\xi}\frac{F_G-F_M}{F_L}\right)^{\frac{k-\sigma+1}{1-\sigma}}}{(\bar{a}/a_L)^k-1}$$

This is an expression in exogenous variables, a_L, ξ which can be expressed in a_L and exogenous variables and $\epsilon\kappa$, thus showing that $\epsilon\kappa$ ultimately depends only on a_L.

From the definition of the probability density function of the Pareto distribution:

$$\begin{aligned}
\mu &\equiv \int_0^{\bar{a}} ag(a)da = \int_0^{\bar{a}} \frac{aka^{k-1}}{\bar{a}^k}da \\
&= \frac{k}{\bar{a}^k}\int_0^{\bar{a}} a^k da = \frac{k}{\bar{a}^k}\left[\frac{a^{k+1}}{k+1}\right]_0^{\bar{a}} \\
&= \frac{k}{\bar{a}^k}\frac{\bar{a}^{k+1}}{k+1} = k\bar{a}/(k+1).
\end{aligned}$$

Innovation Incentives

$$\begin{aligned}
b_t F_I &= \int_{a_L}^{\bar{a}} (1-\psi)\epsilon E_h\left[v_L(a')\right] g(a_f)da_f \\
&+ \int_0^{a_L} \left(v_L(a) - b_t F_L\right) g(a)da + \int_{a_M}^{a_E} \left(\theta v_L(a) - b_t F_E\right) g(a)da \\
&+ \int_{a_G}^{a_M} \left(\psi E_f\left[v_L(a')\right] - b_t F_M\right) g(a_h)da_h + \int_0^{a_G} \left(v_L(a) - b_t F_G\right) g(a)da.
\end{aligned}$$

I group all fixed costs on the left-hand-side

$$\begin{aligned}
& b_t F_I + b_t F_L G(a_L) + b_t F_E (G(a_E) - G(a_M)) \\
& + b_t F_M (G(a_M) - G(a_G)) + b_t F_G G(a_G) \\
= & \int_{a_L}^{\bar{a}} (1-\psi)\epsilon E_h\left[v_L(a')\right] g(a_f)da_f \\
& + \int_0^{a_L} v_L(a)g(a)da + \int_{a_M}^{a_E} \theta v_L(a)g(a)da \\
& + \int_{a_G}^{a_M} \psi E_f\left[v_L(a')\right] g(a_h)da_h + \int_0^{a_G} v_L(a)g(a)da,
\end{aligned}$$

divide by $G(a_L)$ and define

$$F_x \equiv \frac{F_I}{G(a_L)} + F_L + F_E\frac{G(a_E) - G(a_M)}{G(a_L)} + F_M\frac{G(a_M) - G(a_G)}{G(a_L)} + F_G\frac{G(a_G)}{G(a_L)}.$$

The innovation incentives condition becomes

$$\begin{aligned}
b_t F_x &= \int_{a_L}^{\bar{a}} (1-\psi)\epsilon E_h\left[v_L(a')\right] g(a_f)da_f \\
&+ \int_0^{a_L} v_L(a)g(a)da + \int_{a_M}^{a_E} \theta v_L(a)g(a)da \\
&+ \int_{a_G}^{a_M} \psi E_f\left[v_L(a')\right] g(a_h)da_h + \int_0^{a_G} v_L(a)g(a)da,
\end{aligned}$$

In the first integral I substitute for $\epsilon = \frac{G(a_M)-G(a_G)}{1-G(a_L)}$ to obtain

$$\begin{aligned}
\int_{a_L}^{\bar{a}} (1-\psi)\epsilon E_h\left[v_L(a')\right] g(a_f)da_f &= (1-\psi)\frac{G(a_M)-G(a_G)}{1-G(a_L)}\int_{a_L}^{\bar{a}} E_h\left[v_L(a')\right]\frac{1-G(a_L)}{1-G(a_L)}g(a_f)da_f \\
&= (1-\psi)\left(G(a_M)-G(a_G)\right)\int_{a_L}^{\bar{a}} E_h\left[v_L(a')\right]\frac{g(a_f)}{1-G(a_L)}da_f \\
&= (1-\psi)\left(G(a_M)-G(a_G)\right)E_f\left[E_h\left[v_L(a')\right]\right] \\
&= (1-\psi)\left(G(a_M)-G(a_G)\right)E_h\left[E_f\left[v_L(a')\right]\right]
\end{aligned}$$

The fourth integral I rewrite

$$\begin{aligned}
\int_{a_G}^{a_M} \psi E_f\left[v_L(a')\right] g(a_h)da_h &= \psi\int_{a_G}^{a_M} E_f\left[v_L(a')\right]\frac{G(a_M)-G(a_G)}{G(a_M)-G(a_G)}g(a_h)da_h \\
&= \psi\left(G(a_M)-G(a_G)\right)E_h\left[E_f\left[v_L(a')\right]\right]
\end{aligned}$$

The innovation incentives condition therefore becomes

$$\begin{aligned}
b_tF_x &= \int_0^{a_L} v_L(a)g(a)da + \int_{a_M}^{a_E}\theta v_L(a)g(a)da \\
&\quad + \left(G(a_M)-G(a_G)\right)E_h\left[E_f\left[v_L(a')\right]\right] \\
&\quad + \int_0^{a_G} v_L(a)g(a)da.
\end{aligned}$$

I can rewrite

$$\begin{aligned}
E_h\left[E_f\left[v_L(a')\right]\right] &= \int_{a_G}^{a_M} E_f\left[v_L(a')\right]\frac{g(a_h)}{G(a_M)-G(a_G)}da_h \\
&= \int_{a_G}^{a_M} \xi v_L(a_h)\frac{g(a_h)}{G(a_M)-G(a_G)}da_h.
\end{aligned}$$

Substituting in the innovation incentives condition yields

$$\begin{aligned}
b_tF_x &= \int_0^{a_L} v_L(a)\frac{g(a)}{G(a_L)}da + \int_{a_M}^{a_E}\theta v_L(a)\frac{g(a)}{G(a_L)}da \\
&\quad + \int_{a_G}^{a_M}\xi v_L(a_h)\frac{g(a_h)}{G(a_L)}da_h + \int_0^{a_G} v_L(a)\frac{g(a)}{G(a_L)}da.
\end{aligned}$$

Next I substitute for $v_L(a)$:

$$\begin{aligned}
b_tF_x &= \int_0^{a_L}\frac{\pi_L(a)}{r-g}\frac{g(a)}{G(a_L)}da + \theta\int_{a_M}^{a_E}\frac{\pi_L(a)}{r-g}\frac{g(a)}{G(a_L)}da \\
&\quad + \xi\int_{a_G}^{a_M}\frac{\pi_L(a_h)}{r-g}\frac{g(a_h)}{G(a_L)}da_h + \int_0^{a_G}\frac{\pi_L(a)}{r-g}\frac{g(a)}{G(a_L)}da
\end{aligned}$$

and for profits from (3)

$$b_t F_x = \frac{\delta P_t^{\sigma-1} C_t}{r-g} \left(\begin{array}{c} \int_0^{a_L} a^{1-\sigma} \frac{g(a)}{G(a_L)} da + \theta \int_{a_M}^{a_E} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \\ + \xi \int_{a_G}^{a_M} a_h^{1-\sigma} \frac{g(a_h)}{G(a_L)} da_h + \int_0^{a_G} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \end{array} \right).$$

Defining the expression in brackets as Δ, I obtain

$$b_t F_x = \frac{\delta P_t^{\sigma-1} C_t}{r-g} \Delta.$$

This is the innovation incentives condition (19).

The Price Index

$$\begin{aligned} P_t^{1-\sigma} &= \int_0^{m_t^c} p_t(\omega)^{1-\sigma} d\omega \\ &= \int_0^{a_L} p_L(a)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da \\ &\quad + \int_{a_M}^{a_E} p_E(a)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da \\ &\quad + \int_{a_G}^{a_M} \left(\int_{a_L}^{\bar{a}} p_L(a')^{1-\sigma} \frac{g(a_f)}{1-G(a_L)} da_f \right) m_t \frac{g(a_h)}{G(a_L)} da_h \\ &\quad + \int_0^{a_G} p_L(a)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da. \end{aligned}$$

I substitute for prices

$$\begin{aligned} P_t^{1-\sigma} &= \int_0^{a_L} \left(\frac{\sigma}{\sigma-1} a \right)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da \\ &\quad + \int_{a_M}^{a_E} \left(\frac{\sigma}{\sigma-1} \tau a \right)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da \\ &\quad + \int_{a_G}^{a_M} \left(\int_{a_L}^{\bar{a}} \left(\frac{\sigma}{\sigma-1} a' \right)^{1-\sigma} \frac{g(a_f)}{1-G(a_L)} da_f \right) m_t \frac{g(a_h)}{G(a_L)} da_h \\ &\quad + \int_0^{a_G} \left(\frac{\sigma}{\sigma-1} a \right)^{1-\sigma} m_t \frac{g(a)}{G(a_L)} da. \end{aligned}$$

After substituting for a' and rearranging, I obtain

$$P_t^{1-\sigma} = m_t \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left(\begin{array}{c} \int_0^{a_L} a^{1-\sigma} \frac{g(a)}{G(a_L)} da + \theta \int_{a_M}^{a_E} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \\ + \int_{a_G}^{a_M} \left(\int_{a_L}^{\bar{a}} \left(\frac{a_f}{\mu} a_h \right)^{1-\sigma} \frac{g(a_f)}{1-G(a_L)} da_f \right) \frac{g(a_h)}{G(a_L)} da_h \\ + \int_0^{a_G} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \end{array} \right).$$

Since

$$\int_{a_L}^{\bar{a}} \left(\frac{a_f}{\mu}a_h\right)^{1-\sigma} \frac{g(a_f)da_f}{1-G(a_L)} = a_h^{1-\sigma}E_f\left[(a_f/\mu)^{1-\sigma}\right] = \xi a_h^{1-\sigma},$$

I obtain

$$\begin{aligned} P_t^{1-\sigma} &= m_t\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\begin{array}{c} \int_0^{a_L} a^{1-\sigma}\frac{g(a)}{G(a_L)}da + \theta\int_{a_M}^{a_E} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \\ +\xi\int_{a_G}^{a_M} a_h^{1-\sigma}\frac{g(a_h)}{G(a_L)}da_h + \int_0^{a_G} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \end{array}\right) \\ &= m_t\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\Delta, \end{aligned}$$

which is equation (20) in the main text.

Steady State Labor Market Clearing

I first calculate labor involved in production. Labor involved in production at t is

$$L_{Pt} = m_t\left(\begin{array}{c} \int_0^{a_L} ax_{Lt}\frac{g(a)}{G(a_L)}da + \int_{a_M}^{a_E}\tau ax_{Et}\frac{g(a)}{G(a_L)}da \\ +\int_{a_G}^{a_M}\left(\int_{a_L}^{\bar{a}} a'x_t'\frac{g(a_f)da_f}{1-G(a_L)}\right)\frac{g(a_h)}{G(a_L)}da_h + \int_0^{a_G} ax_{Lt}\frac{g(a)}{G(a_L)}da \end{array}\right).$$

To produce a variety for the local market a firm needs to use ax_{Lt} labor units. Using (1) yields $ax_{Lt} = ap_L(a)^{-\sigma}P_t^{\sigma-1}C_t$, where I have used aggregate consumption $C_t = c_tL_t$. I substitute for the optimal price $p_L(a) = \frac{\sigma}{\sigma-1}a$ to obtain $ax_{Lt} = a^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_t^{\sigma-1}C_t$. For exporting, a firm produces with $\tau ax_{Et} = \tau ap_E(a)^{-\sigma}P_t^{\sigma-1}C_t$ labor units, where I use (1) to substitute for demand. Substituting for $p_E(a) = \frac{\sigma}{\sigma-1}\tau a$ yields $\tau ax_{Et} = \tau^{1-\sigma}a^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_t^{\sigma-1}C_t = \theta a^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_t^{\sigma-1}C_t$, where I have used $\theta \equiv \tau^{1-\sigma}$. The local firms that were taken over by a foreign firm need $a'x_t'$ units of labor. Using (1) to substitute for demand yields $a'x_t' = a'p_L(a')^{-\sigma}P_t^{\sigma-1}C_t$. I substitute for the optimal price $p_L(a') = \frac{\sigma}{\sigma-1}a'$ to obtain $a'x_{Lt}' = (a')^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_t^{\sigma-1}C_t = \left(\frac{a_f}{\mu}a_h\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_t^{\sigma-1}C_t$. Going back to the expression for L_{Pt}, I obtain

$$\begin{aligned} L_{Pt} &= m_t\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_t^{\sigma-1}C_t\left(\begin{array}{c} \int_0^{a_L} a^{1-\sigma}\frac{g(a)}{G(a_L)}da + \theta\int_{a_M}^{a_E} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \\ +\int_{a_G}^{a_M}\left(\int_{a_L}^{\bar{a}}\left(\frac{a_f}{\mu}a_h\right)^{1-\sigma}\frac{g(a_f)da_f}{1-G(a_L)}\right)\frac{g(a_h)}{G(a_L)}da_h \\ +\int_0^{a_G} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \end{array}\right) \\ &= m_t\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_t^{\sigma-1}C_t\left(\begin{array}{c} \int_0^{a_L} a^{1-\sigma}\frac{g(a)}{G(a_L)}da + \theta\int_{a_M}^{a_E} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \\ +\int_{a_G}^{a_M} a_h^{1-\sigma}\left(\int_{a_L}^{\bar{a}}(a_f/\mu)^{1-\sigma}\frac{g(a_f)da_f}{1-G(a_L)}\right)\frac{g(a_h)}{G(a_L)}da_h \\ +\int_0^{a_G} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \end{array}\right). \end{aligned}$$

I know that $\xi \equiv E_f\left[(a_f/\mu)^{1-\sigma}\right]$, therefore

$$L_{Pt} = m_t\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_t^{\sigma-1}C_t\left(\begin{array}{c} \int_0^{a_L} a^{1-\sigma}\frac{g(a)}{G(a_L)}da + \theta\int_{a_M}^{a_E} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \\ +\xi\int_{a_G}^{a_M} a_h^{1-\sigma}\frac{g(a_h)}{G(a_L)}da_h + \int_0^{a_G} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \end{array}\right).$$

I substitute for Δ

$$L_{Pt} = m_t \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} P_t^{\sigma-1} C_t \Delta,$$

and then for $P_t^{1-\sigma} = m_t \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \Delta$ to obtain

$$\begin{aligned} L_{Pt} &= \frac{m_t \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} C_t \Delta}{m_t \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \Delta} \\ &= \frac{\sigma - 1}{\sigma} C_t. \end{aligned}$$

The full employment condition $L_t = L_{Pt} + L_{It}$ implies that $L_t = \frac{\sigma-1}{\sigma} C_t + L_{It}$, which in turn leads to equation (21) in the main text

$$C_t = L_t + \frac{C_t}{\sigma} - L_{It}.$$

To show that C_t/σ is aggregate income from profits, I integrate over the profits of all companies originating from a country and denote that value as Λ:

$$\begin{aligned} \Lambda \equiv & \int_0^{a_L} \pi_{Lt}(a) m_t \frac{g(a)}{G(a_L)} da + \int_{a_M}^{a_E} \pi_{Et}(a) m_t \frac{g(a)}{G(a_L)} da \\ & + \int_{a_G}^{a_M} \left(\int_{a_L}^{\bar{a}} \pi_{Lt}(a') m_t \frac{g(a_f)}{1 - G(a_L)} da_f \right) \frac{g(a_h)}{G(a_L)} da_h \\ & + \int_0^{a_G} \pi_{Lt}(a) m_t \frac{g(a)}{G(a_L)} da. \end{aligned}$$

I substitute for profits

$$\begin{aligned} \Lambda = & \int_0^{a_L} \delta a^{1-\sigma} P_t^{\sigma-1} C_t m_t \frac{g(a)}{G(a_L)} da + \theta \int_{a_M}^{a_E} \delta a^{1-\sigma} P_t^{\sigma-1} C_t m_t \frac{g(a)}{G(a_L)} da \\ & + \int_{a_G}^{a_M} \left(\int_{a_L}^{\bar{a}} \delta \left(a'\right)^{1-\sigma} P_t^{\sigma-1} C_t m_t \frac{g(a_f)}{1 - G(a_L)} da_f \right) \frac{g(a_h)}{G(a_L)} da_h \\ & + \int_0^{a_G} \delta a^{1-\sigma} P_t^{\sigma-1} C_t m_t \frac{g(a)}{G(a_L)} da, \end{aligned}$$

and then for a'

$$\begin{aligned} \Lambda &= \delta P_t^{\sigma-1} C_t m_t \left(\begin{array}{c} \int_0^{a_L} a^{1-\sigma} \frac{g(a)}{G(a_L)} da + \theta \int_{a_M}^{a_E} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \\ + \int_{a_G}^{a_M} \left(\int_{a_L}^{\bar{a}} \left(\frac{a_f}{\mu} a_h \right)^{1-\sigma} \frac{g(a_f)}{1-G(a_L)} da_f \right) \frac{g(a_h)}{G(a_L)} da_h + \int_0^{a_G} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \end{array} \right) \\ &= \delta P_t^{\sigma-1} C_t m_t \left(\begin{array}{c} \int_0^{a_L} a^{1-\sigma} \frac{g(a)}{G(a_L)} da + \theta \int_{a_M}^{a_E} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \\ + \int_{a_G}^{a_M} a_h^{1-\sigma} \left(\int_{a_L}^{\bar{a}} (a_f/\mu)^{1-\sigma} \frac{g(a_f)}{1-G(a_L)} da_f \right) \frac{g(a_h)}{G(a_L)} da_h + \int_0^{a_G} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \end{array} \right). \end{aligned}$$

I know that $\xi \equiv E_f\left[(a_f/\mu)^{1-\sigma}\right]$, therefore

$$\Lambda = \delta P_t^{\sigma-1} C_t m_t \begin{pmatrix} \int_0^{a_L} a^{1-\sigma} \frac{g(a)}{G(a_L)} da + \theta \int_{a_M}^{a_E} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \\ +\xi \int_{a_G}^{a_M} a^{1-\sigma} \frac{g(a_h)}{G(a_L)} da_h + \int_0^{a_G} a^{1-\sigma} \frac{g(a)}{G(a_L)} da \end{pmatrix}.$$

Substituting for Δ yields

$$\Lambda = \delta P_t^{\sigma-1} C_t m_t \Delta.$$

Substituting for $P_t^{1-\sigma} = m_t \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Delta$ yields

$$\Lambda = \frac{\delta C_t m_t \Delta}{m_t \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Delta}.$$

From $\delta \equiv (\sigma-1)^{\sigma-1}\sigma^{-\sigma}$, I obtain $\delta / \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} = 1/\sigma$, therefore

$$\Lambda = \frac{C_t}{\sigma}.$$

Solving the Model

I rearrange $L_{It} = \dot{m}_t b_t F_x$ to obtain $\dot{m}_t = \frac{L_{It}}{b_t F_x}$. Therefore

$$\begin{aligned} \gamma &\equiv \frac{\dot{m}_t}{m_t} = \frac{L_{It}}{b_t F_x m_t} \\ &= \frac{L_{It}(1+\lambda)^{\phi} m_t^{\phi}}{F_x m_t} \\ &= \frac{L_{It}}{m_t^{1-\phi}} \frac{(1+\lambda)^{\phi}}{F_x}. \end{aligned}$$

Since γ is constant in steady state, L_{It} must grow at the same rate as $m_t^{1-\phi}$, so $n = (1-\phi)\dot{m}_t/m_t$, and therefore $\gamma = \frac{n}{1-\phi}$.

Using the expression for profits

$$\pi_{Lt} = \delta \left(\frac{a(\omega)}{P_t}\right)^{1-\sigma} C_t,$$

and substituting for the price index from (20) yields

$$\begin{aligned} \pi_{Lt} &= \frac{\delta a^{1-\sigma} C_t}{m_t \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Delta} \\ &= \frac{(\sigma-1)^{\sigma-1}\sigma^{-\sigma} a^{1-\sigma} C_t}{m_t \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Delta} \\ &= \sigma^{-1} a^{1-\sigma} \Delta^{-1} c_t L_t / m_t. \end{aligned}$$

This is to show that the relevant market size for variety ω is L_t/m_t.

To find (23), I use (7):

$$a_L^{1-\sigma} = \frac{F_L}{1-(1-\psi)\epsilon\kappa}\frac{b_t(r-g)}{\delta P_t^{\sigma-1}C_t},$$

substitute for the price index from (20) and for δ:

$$a_L^{1-\sigma} = \frac{F_L}{1-(1-\psi)\epsilon\kappa}\frac{b_t(r-g)m_t\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\Delta}{(\sigma-1)^{\sigma-1}\sigma^{-\sigma}C_t},$$

substitute for b_t from (2):

$$a_L^{1-\sigma} = \frac{F_L}{1-(1-\psi)\epsilon\kappa}\frac{(r-g)m_t\sigma\Delta}{(1+\lambda)^{\phi}m_t^{\phi}C_t},$$

multiply the right-hand-side by L_t/L_t

$$a_L^{1-\sigma} = \frac{F_L}{1-(1-\psi)\epsilon\kappa}\frac{(r-g)m_t^{1-\phi}\sigma\Delta}{(1+\lambda)^{\phi}C_t}\frac{L_t}{L_t}$$

and substitute for $z \equiv m_t^{1-\phi}/L_t$ to arrive at

$$a_L^{1-\sigma} = \frac{F_L}{1-(1-\psi)\epsilon\kappa}\frac{(r-g)\sigma\Delta}{(1+\lambda)^{\phi}C_t}zL_t$$

or

$$\frac{\Delta}{a_L^{1-\sigma}}\frac{F_L}{1-(1-\psi)\epsilon\kappa} = \frac{(1+\lambda)^{\phi}C_t}{z(r-g)\sigma L_t},$$

which is equation (23) in the main text. In the innovation incentives condition (19), I substitute for the price index from (20):

$$\begin{aligned}
b_tF_x &= \frac{\delta P_t^{\sigma-1}C_t}{r-g}\Delta \\
&= \frac{\delta C_t}{m_t\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\Delta(r-g)}\Delta \\
&= \frac{(\sigma-1)^{\sigma-1}\sigma^{-\sigma}C_t}{m_t\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}(r-g)} \\
&= \frac{C_t}{(r-g)m_t\sigma},
\end{aligned}$$

for b_t from (2):

$$\begin{aligned}
F_x &= \frac{(1+\lambda)^{\phi}m_t^{\phi}C_t}{(r-g)m_t\sigma} \\
&= \frac{(1+\lambda)^{\phi}C_t}{(r-g)m_t^{1-\phi}\sigma}\frac{L_t}{L_t}
\end{aligned}$$

and for $z \equiv m_t^{1-\phi}/L_t$ to obtain equation (24) in the main text:

$$F_x = \frac{(1+\lambda)^{\phi} C_t}{z(r-g)\sigma L_t}.$$

Next I find an expression for Δ in one unknown a_L. Using the properties of the Pareto distribution

$$\frac{g(a)}{G(a_L)} = \frac{\left(ka^{k-1}/\bar{a}^k\right)}{\left(a_L^k/\bar{a}^k\right)} = \frac{ka^{k-1}}{a_L^k}$$

and the definition for Δ gives:

$$\begin{aligned}
\Delta &\equiv \int_0^{a_L} a^{1-\sigma}\frac{g(a)}{G(a_L)}da + \theta\int_{a_M}^{a_E} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \\
&\quad +\xi \int_{a_G}^{a_M} a_h^{1-\sigma}\frac{g(a_h)}{G(a_L)}da_h + \int_0^{a_G} a^{1-\sigma}\frac{g(a)}{G(a_L)}da \\
&= \int_0^{a_L} a^{1-\sigma}\frac{ka^{k-1}}{a_L^k}da + \theta\int_{a_M}^{a_E} a^{1-\sigma}\frac{ka^{k-1}}{a_L^k}da \\
&\quad +\xi \int_{a_G}^{a_M} a_h^{1-\sigma}\frac{ka^{k-1}}{a_L^k}da_h + \int_0^{a_G} a^{1-\sigma}\frac{ka^{k-1}}{a_L^k}da.
\end{aligned}$$

The previous assumption that $k > \sigma - 1$ guarantees that the integrals converge and Δ is finite. Lower values of k would result in explosive profits for marginal costs close to zero. Using $\eta \equiv k/(k-\sigma+1)$ and solving the integrals yields

$$\begin{aligned}
\Delta &= \frac{\eta}{a_L^k}\left[a^{k-\sigma+1}\right]_0^{a_L} + \theta\frac{\eta}{a_L^k}\left[a^{k-\sigma+1}\right]_{a_M}^{a_E} \\
&\quad +\xi\frac{\eta}{a_L^k}\left[a_h^{k-\sigma+1}\right]_{a_G}^{a_M} + \frac{\eta}{a_L^k}\left[a^{k-\sigma+1}\right]_0^{a_G} \\
&= \eta\frac{a_L^{k-\sigma+1}}{a_L^k} + \eta\frac{\theta}{a_L^k}\left(a_E^{k-\sigma+1} - a_M^{k-\sigma+1}\right) \\
&\quad +\eta\frac{\xi}{a_L^k}\left(a_M^{k-\sigma+1} - a_G^{k-\sigma+1}\right) + \eta\frac{a_G^{k-\sigma+1}}{a_L^k} \\
&= \eta a_L^{1-\sigma} + \eta a_L^{1-\sigma}\theta\left(\left(a_E/a_L\right)^{k-\sigma+1} - \left(a_M/a_L\right)^{k-\sigma+1}\right) \\
&\quad +\eta a_L^{1-\sigma}\xi\left(\left(a_M/a_L\right)^{k-\sigma+1} - \left(a_G/a_L\right)^{k-\sigma+1}\right) \\
&\quad +\eta a_L^{1-\sigma}\left(a_G/a_L\right)^{k-\sigma+1} \\
&= \eta a_L^{1-\sigma}\begin{pmatrix} 1+\theta\left(a_E/a_L\right)^{k-\sigma+1} - \theta\left(a_M/a_L\right)^{k-\sigma+1} \\ +\xi\left(a_M/a_L\right)^{k-\sigma+1} - \xi\left(a_G/a_L\right)^{k-\sigma+1} \\ +\left(a_G/a_L\right)^{k-\sigma+1} \end{pmatrix} \\
&= \eta a_L^{1-\sigma}\begin{pmatrix} 1+\theta\left(a_E/a_L\right)^{k-\sigma+1} + (\xi-\theta)\left(a_M/a_L\right)^{k-\sigma+1} \\ +(1-\xi)\left(a_G/a_L\right)^{k-\sigma+1} \end{pmatrix}.
\end{aligned}$$

To find $(a_E/a_L)^{k-\sigma+1}$, I use (7) and (10):

$$\begin{aligned}
\frac{a_E^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{\theta}\frac{F_E}{F_L} \\
\left(\frac{a_E}{a_L}\right)^{k-\sigma+1} &= \left(\frac{1-(1-\psi)\epsilon\kappa}{\theta}\frac{F_E}{F_L}\right)^{\frac{k-\sigma+1}{1-\sigma}}.
\end{aligned}$$

To find $(a_M/a_L)^{k-\sigma+1}$, I use (7) and (11):

$$\begin{aligned}
\frac{a_M^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{\psi\xi-\theta}\frac{F_M-F_E}{F_L} \\
\left(\frac{a_M}{a_L}\right)^{k-\sigma+1} &= \left(\frac{1-(1-\psi)\epsilon\kappa}{\psi\xi-\theta}\frac{F_M-F_E}{F_L}\right)^{\frac{k-\sigma+1}{1-\sigma}}.
\end{aligned}$$

To find $(a_G/a_L)^{k-\sigma+1}$, I use (7) and (12):

$$\begin{aligned}
\frac{a_G^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{1-\psi\xi}\frac{F_G-F_M}{F_L} \\
\left(\frac{a_G}{a_L}\right)^{k-\sigma+1} &= \left(\frac{1-(1-\psi)\epsilon\kappa}{1-\psi\xi}\frac{F_G-F_M}{F_L}\right)^{\frac{k-\sigma+1}{1-\sigma}}.
\end{aligned}$$

I define $\beta \equiv k/(\sigma-1)$ and write $\frac{k-\sigma+1}{1-\sigma} = 1-\frac{k}{1-\sigma} = 1-\beta$. I substitute for the above three ratios of the threshold marginal costs into the expression for Δ to obtain

$$\Delta \equiv \eta a_L^{1-\sigma}\left(\begin{array}{c} 1+\theta\left(\frac{1-(1-\psi)\epsilon\kappa}{\theta}\frac{F_E}{F_L}\right)^{1-\beta}+(\xi-\theta)\left(\frac{1-(1-\psi)\epsilon\kappa}{\psi\xi-\theta}\frac{F_M-F_E}{F_L}\right)^{1-\beta} \\ +(1-\xi)\left(\frac{1-(1-\psi)\epsilon\kappa}{1-\psi\xi}\frac{F_G-F_M}{F_L}\right)^{1-\beta} \end{array}\right).$$

This is equation (25) in the main text where $q_1()$ is the expression in brackets and is a function of a_L and exogenous variables only.

Finding a_L

From (18),

$$F_x \equiv \frac{F_I}{G(a_L)} + F_L + F_E\frac{G(a_E)-G(a_M)}{G(a_L)} + F_M\frac{G(a_M)-G(a_G)}{G(a_L)} + F_G\frac{G(a_G)}{G(a_L)}.$$

I substitute for the cumulative distribution functions

$$
\begin{aligned}
F_x &= \frac{F_I}{(a_L/\bar{a})^k} + F_L + F_E \frac{(a_E/\bar{a})^k - (a_M/\bar{a})^k}{(a_L/\bar{a})^k} \\
&\quad + F_M \frac{(a_M/\bar{a})^k - (a_G/\bar{a})^k}{(a_L/\bar{a})^k} + F_G \frac{(a_G/\bar{a})^k}{(a_L/\bar{a})^k} \\
&= F_I \left(\bar{a}/a_L\right)^k + F_L + F_E \left((a_E/a_L)^k - (a_M/a_L)^k\right) \\
&\quad + F_M \left((a_M/a_L)^k - (a_G/a_L)^k\right) + F_G (a_G/a_L)^k \\
&= F_I \left(\bar{a}/a_L\right)^k + F_L + F_E (a_E/a_L)^k \\
&\quad + (F_M - F_E)(a_M/a_L)^k \\
&\quad + (F_G - F_M)(a_G/a_L)^k.
\end{aligned}
$$

From (7) and (10):

$$
\begin{aligned}
\frac{a_E^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{\theta} \frac{F_E}{F_L} \\
\left(\frac{a_E}{a_L}\right)^k &= \left(\frac{1-(1-\psi)\epsilon\kappa}{\theta} \frac{F_E}{F_L}\right)^{\frac{k}{1-\sigma}} = \left(\frac{1-(1-\psi)\epsilon\kappa}{\theta} \frac{F_E}{F_L}\right)^{-\beta}.
\end{aligned}
$$

From (7) and (11):

$$
\begin{aligned}
\frac{a_M^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{\psi\xi - \theta} \frac{F_M - F_E}{F_L} \\
\left(\frac{a_M}{a_L}\right)^k &= \left(\frac{1-(1-\psi)\epsilon\kappa}{\psi\xi - \theta} \frac{F_M - F_E}{F_L}\right)^{\frac{k}{1-\sigma}} = \left(\frac{1-(1-\psi)\epsilon\kappa}{\psi\xi - \theta} \frac{F_M - F_E}{F_L}\right)^{-\beta}.
\end{aligned}
$$

From (7) and (12):

$$
\begin{aligned}
\frac{a_G^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{1 - \psi\xi} \frac{F_G - F_M}{F_L} \\
\left(\frac{a_G}{a_L}\right)^k &= \left(\frac{1-(1-\psi)\epsilon\kappa}{1 - \psi\xi} \frac{F_G - F_M}{F_L}\right)^{\frac{k}{1-\sigma}} = \left(\frac{1-(1-\psi)\epsilon\kappa}{1 - \psi\xi} \frac{F_G - F_M}{F_L}\right)^{-\beta}.
\end{aligned}
$$

I substitute for the above three ratios of the threshold marginal costs into the expression for F_x to obtain

$$
\begin{aligned}
F_x &= F_I \left(\bar{a}/a_L\right)^k + F_L + F_E \left(\frac{1-(1-\psi)\epsilon\kappa}{\theta} \frac{F_E}{F_L}\right)^{-\beta} \\
&\quad + (F_M - F_E) \left(\frac{1-(1-\psi)\epsilon\kappa}{\psi\xi - \theta} \frac{F_M - F_E}{F_L}\right)^{-\beta} \\
&\quad + (F_G - F_M) \left(\frac{1-(1-\psi)\epsilon\kappa}{1 - \psi\xi} \frac{F_G - F_M}{F_L}\right)^{-\beta}.
\end{aligned}
$$

I rewrite the above as $F_x = q_2()$, where $q_2()$ is the expression on the right-hand-side and is a function of a_L and exogenous variables only.

To find C_t using $C_t = L_t + \frac{C_t}{\sigma} - L_{It}$, I first find L_{It} and C_t/σ. I solve (22) for $\dot{m}_t$

$$\dot{m}_t = \frac{L_{It}}{b_t F_x}$$

and divide both sides by m_t to obtain

$$\gamma \equiv \frac{\dot{m}_t}{m_t} = \frac{L_{It}}{b_t F_x m_t}.$$

Then substituting for b_t from (2) yields

$$\gamma = \frac{(1+\lambda)^{\phi} m_t^{\phi} L_{It}}{F_x m_t}.$$

or

$$\begin{aligned} L_{It} &= \frac{F_x m_t \gamma}{(1+\lambda)^{\phi} m_t^{\phi}} \\ &= \frac{F_x m_t^{1-\phi} \gamma}{(1+\lambda)^{\phi}}. \end{aligned}$$

I use $z \equiv m_t^{1-\phi}/L_t$ to arrive at an equation for R&D labor (equation (27) in the main text)

$$L_{It} = \frac{F_x z L_t}{(1+\lambda)^{\phi}} \gamma.$$

To solve for C_t/σ, I substitute for the price index (20) in the innovation incentives condition (19)

$$\begin{aligned} b_t F_x &= \frac{\delta P_t^{\sigma-1} C_t}{r-g} \Delta \\ &= \frac{\delta C_t}{(r-g)\, m_t \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Delta} \Delta \\ &= \frac{(\sigma-1)^{\sigma-1} \sigma^{-\sigma} C_t}{(r-g)\, m_t \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}} \\ &= \frac{C_t}{(r-g)\, m_t \sigma} \end{aligned}$$

and rewrite

$$\frac{C_t}{\sigma} = b_t F_x \,(r-g)\, m_t.$$

Substituting for b_t using (2) yields

$$\frac{C_t}{\sigma} = \frac{F_x(r-g)}{(1+\lambda)^{\phi}} m_t^{1-\phi},$$

and then using $z \equiv m_t^{1-\phi}/L_t$ yields

$$\frac{C_t}{\sigma} = (r-g)\frac{F_x z L_t}{(1+\lambda)^{\phi}}.$$

This is equation (28) in the main text.

I substitute for L_{It} and C_t/σ in (21) in order to solve for C_t

$$\begin{aligned} C_t &= L_t + \frac{C_t}{\sigma} - L_{It} \\ &= L_t + (r-g)\frac{F_x z L_t}{(1+\lambda)^{\phi}} - \frac{F_x z L_t}{(1+\lambda)^{\phi}}\gamma \\ &= L_t\left(1 + \frac{(r-g)F_x z}{(1+\lambda)^{\phi}} - \frac{F_x z}{(1+\lambda)^{\phi}}\gamma\right), \end{aligned}$$

which is equation (29) in the main text.

To find z, I substitute for C_t from (29) into (24):

$$F_x = \frac{(1+\lambda)^{\phi} C_t}{z(r-g)\sigma L_t}.$$

$$\begin{aligned} F_x &= \frac{(1+\lambda)^{\phi} L_t\left(1 + \frac{(r-g)F_x z}{(1+\lambda)^{\phi}} - \frac{F_x z}{(1+\lambda)^{\phi}}\gamma\right)}{z(r-g)\sigma L_t} \\ zF_x(r-g)\sigma &= (1+\lambda)^{\phi}\left(1 + \frac{zF_x(r-g-\gamma)}{(1+\lambda)^{\phi}}\right) \\ zF_x(r-g)\sigma &= (1+\lambda)^{\phi} + zF_x(r-g-\gamma) \\ zF_x((r-g)\sigma - r + g + \gamma) &= (1+\lambda)^{\phi} \\ z &= \frac{(1+\lambda)^{\phi}}{F_x((r-g)\sigma - (r-g) + \gamma)} \\ z &= \frac{(1+\lambda)^{\phi}}{F_x((\sigma-1)(r-g) + \gamma)}, \end{aligned}$$

which is equation (30) in the main text.

The share of greenfield FDI, Ω, is measured by greenfield FDI value divided by the sum of greenfield FDI and M&A value:

$$\begin{aligned} \Omega &= \frac{b_t F_G \dot{m}_t G(a_G)/G(a_L)}{b_t F_G \dot{m}_t G(a_G)/G(a_L) + b_t F_M \dot{m}_t (G(a_M) - G(a_G))/G(a_L)} \\ &= \frac{F_G G(a_G)}{F_G G(a_G) + F_M (G(a_M) - G(a_G))}. \end{aligned}$$

To measure total FDI, I simply add the values of M&A and greenfield FDI to obtain

$$\mathcal{F}' \equiv b_t \dot{m}_t \frac{F_M G(a_M) + (F_G - F_M)\, G(a_G)}{G(a_L)},$$

where $\mathcal{F}'$ stands for total FDI. I write $\dot{m}_t = \gamma m_t$ and substitute for b_t from (2) to obtain

$$\begin{aligned} \mathcal{F}' &= \frac{\gamma m_t}{(1+\lambda)^{\phi} m_t^{\phi}} \frac{F_M G(a_M) + (F_G - F_M)\, G(a_G)}{G(a_L)} \\ &= \frac{\gamma z L_t}{(1+\lambda)^{\phi}} \frac{F_M G(a_M) + (F_G - F_M)\, G(a_G)}{G(a_L)}. \end{aligned}$$

I am interested in the direction of change of FDI as a result of lower variable costs to trade. Therefore I will calculate a measure of FDI $\mathcal{F} \equiv \frac{(1+\lambda)^{\phi}}{\gamma L_t} \mathcal{F}'$, where $\frac{(1+\lambda)^{\phi}}{\gamma L_t}$ are all values not affected by τ.

To measure economic growth, I evaluate $\dot{u}_t / u_t$. First, in $u_t = C_t / P_t L_t$, I substitute for aggregate expenditure C_t from (29) and for the price index from (20) to obtain

$$\begin{aligned} u_t &= \frac{L_t \left(1 + \frac{(r-g) F_x z}{(1+\lambda)^{\phi}} - \frac{F_x z}{(1+\lambda)^{\phi}} \gamma\right)}{L_t \left(m_t \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \Delta\right)^{\frac{1}{1-\sigma}}} \\ &= \left(1 + \frac{(r-g)\, F_x z}{(1+\lambda)^{\phi}} - \frac{F_x z}{(1+\lambda)^{\phi}} \gamma\right) \left(\frac{\sigma - 1}{\sigma}\right) (m_t \Delta)^{\frac{1}{\sigma - 1}}. \end{aligned}$$

Instantaneous utility grows over time only because the number of varieties grows over time. From $\gamma = n/(1-\phi)$ and the above expression for u_t, it immediately follows that

$$\frac{\dot{u}_t}{u_t} = \frac{n}{(1-\phi)(\sigma - 1)}.$$

The share of workers involved in R&D activities, L_{It}/L_t, can be obtained from the equation for R&D labor (27)

$$\frac{L_{It}}{L_t} = \frac{F_x z}{(1+\lambda)^{\phi}} \gamma$$

and substituting for z from (30):

$$\begin{aligned} \frac{L_{It}}{L_t} &= \frac{F_x}{(1+\lambda)^{\phi}} \frac{(1+\lambda)^{\phi}}{F_x \left((\sigma - 1)(r-g) + \gamma\right)} \gamma \\ &= \frac{\gamma}{(\sigma - 1)(r-g) + \gamma}. \end{aligned}$$

To find a_G, I use (7) and (12):

$$\begin{aligned} \frac{a_G^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1 - (1-\psi)\epsilon\kappa}{1 - \psi\xi} \frac{F_G - F_M}{F_L} \\ a_G &= a_L \left(\frac{1 - (1-\psi)\epsilon\kappa}{1 - \psi\xi} \frac{F_G - F_M}{F_L}\right)^{\frac{1}{1-\sigma}}. \end{aligned}$$

To find a_M, I use (7) and (11):

$$\begin{aligned}\frac{a_M^{1-\sigma}}{a_L^{1-\sigma}} &= \frac{1-(1-\psi)\epsilon\kappa}{\psi\xi-\theta}\frac{F_M-F_E}{F_L} \\ a_M &= a_L\left(\frac{1-(1-\psi)\epsilon\kappa}{\psi\xi-\theta}\frac{F_M-F_E}{F_L}\right)^{\frac{1}{1-\sigma}}.\end{aligned}$$